ÖSSZETARTOTT PLAZMÁK: A NAP ÉS A LABORATÓRIUM

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Why not use, say, air (i.e. ^{14}N)?

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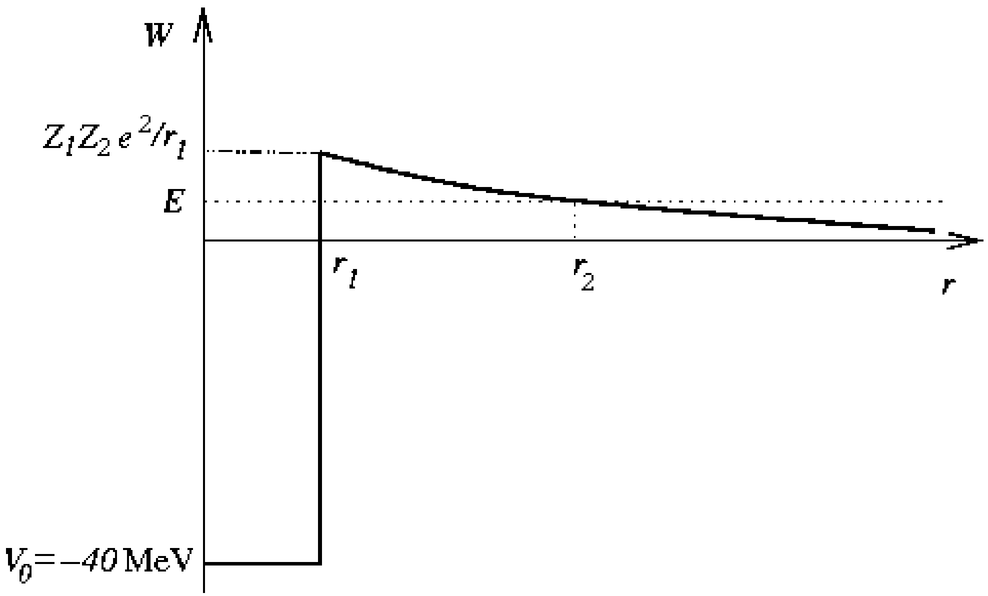

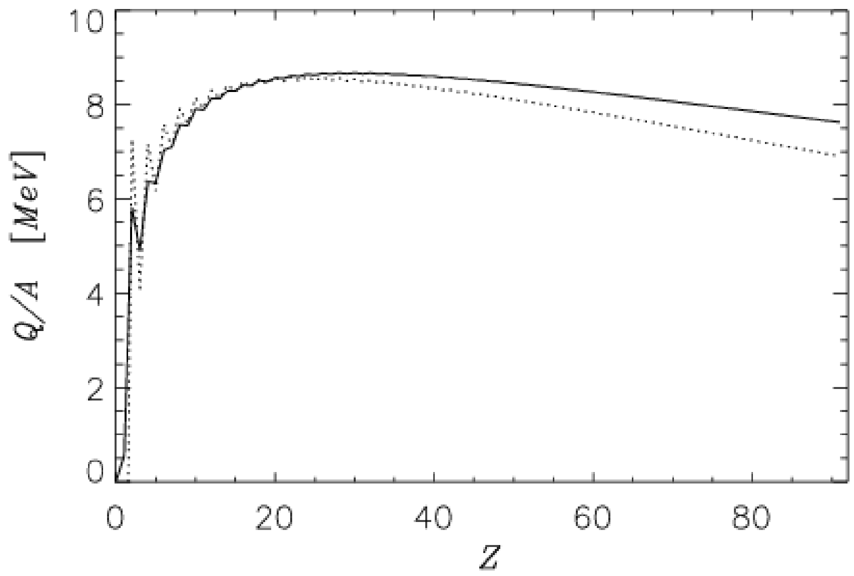

NB Coulomb barrier $\sim Z_1 Z_2 \cdot 10$ MeV (10^{10} K)

⇒ nuclear fusion only due to tunneling, *anywhere* in the cosmos

⇒ steep increase of energy production rate ε_n with T:

$$\varepsilon_n \propto n_1 n_2 T^n \qquad n = 3...30$$

What can we learn form the Sun about controlled fusion:

(2) Plasma temperature and confinement

For efficiency η, for sustained fusion we need $\eta\varepsilon_n > j_{\rm ff}$

where the radiative heat loss in opt.thin plasma is $j_{\rm ff} \propto n_1 n_2 T^{1/2}$

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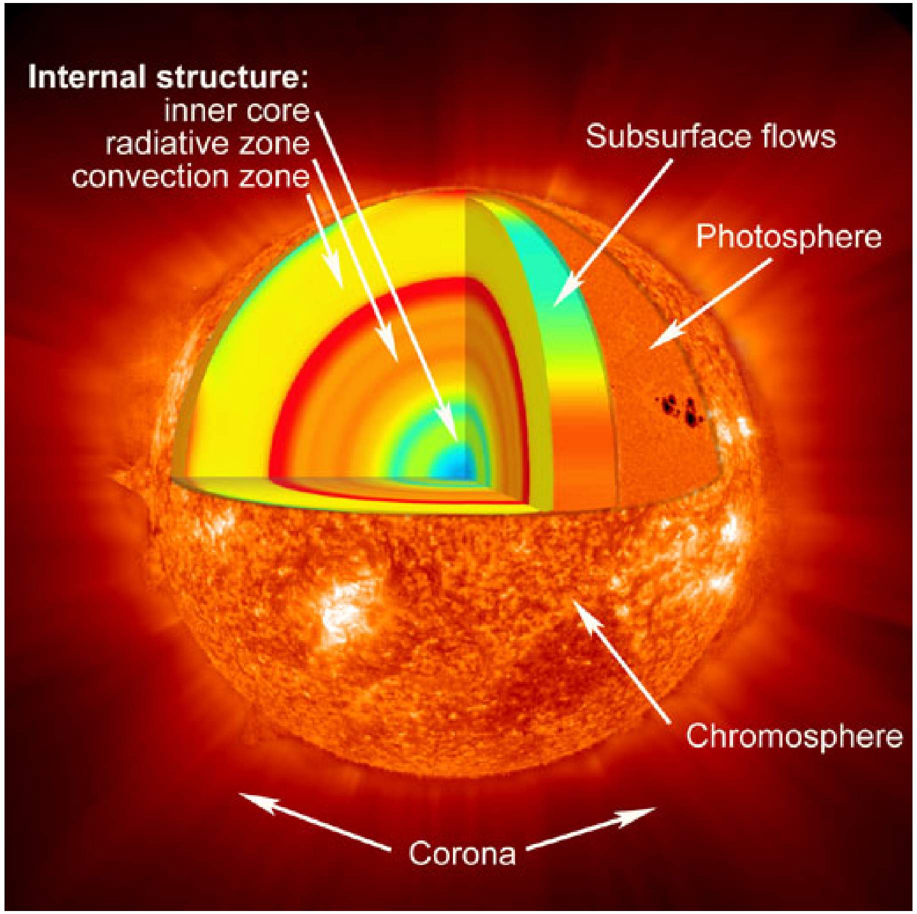
where the radiative heat loss in opt.thin plasma is $j_{\mathrm{ff}} \propto n_1 n_2 T^{1/2}$

$\Rightarrow$ minimum temperature needed for $^2\mathrm{H}(^3\mathrm{H}, n)^3\mathrm{He}$ reaction is $\sim 3 \cdot 10^7$ K;
for finite confinement time even higher, ~100–200 MK.

At the same time, magnetic confinement has technical limits ($\sim 10^5$ G)
$\Rightarrow n/n_{\mathrm{air}} \sim 0.01\ldots$

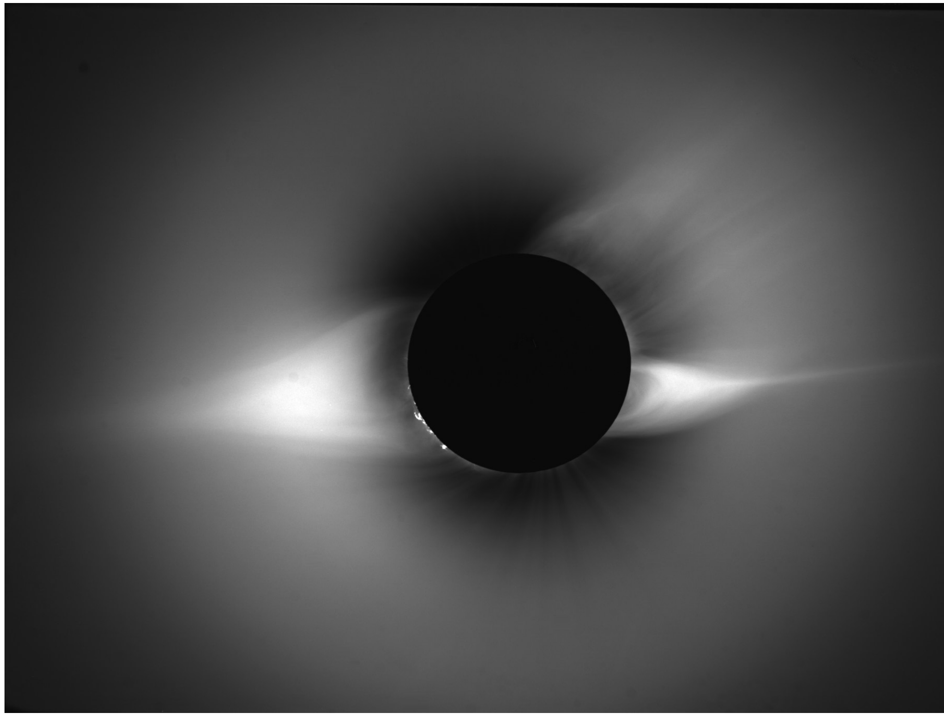
Yet in the solar core we have fusion with $T \sim 1.1$–$1.5 \cdot 10^7$ K!



Layer	Temperature	Density
Corona	10^6 K	10^{-14} g/cm^3
Chromosphere	10 000 K	10^{-10} g/cm^3
Photosphere	6000 K	10^{-6} g/cm^3
Base of conv.zone	1 million K	0,1 g/cm^3
Center	15 million K	150 g/cm^3

In the solar core, the high density fusion plasma is pressure confined — no chance for this in the lab.

Yet magnetic confinement does exist in the Sun: in the corona!

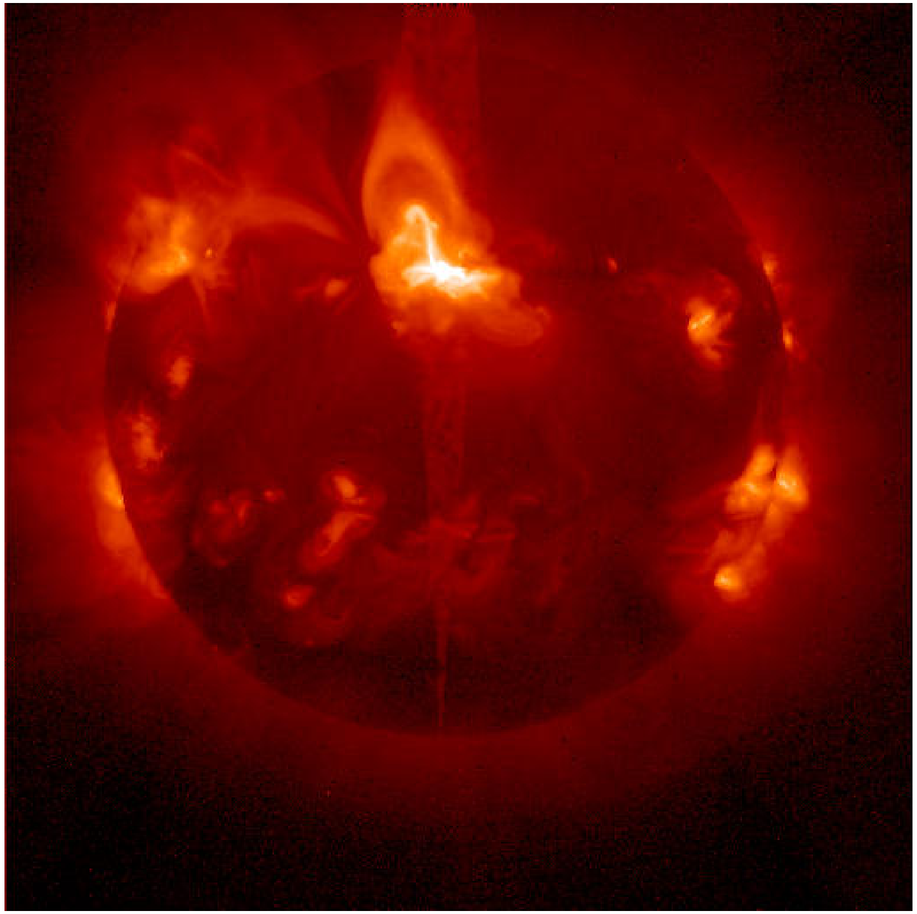


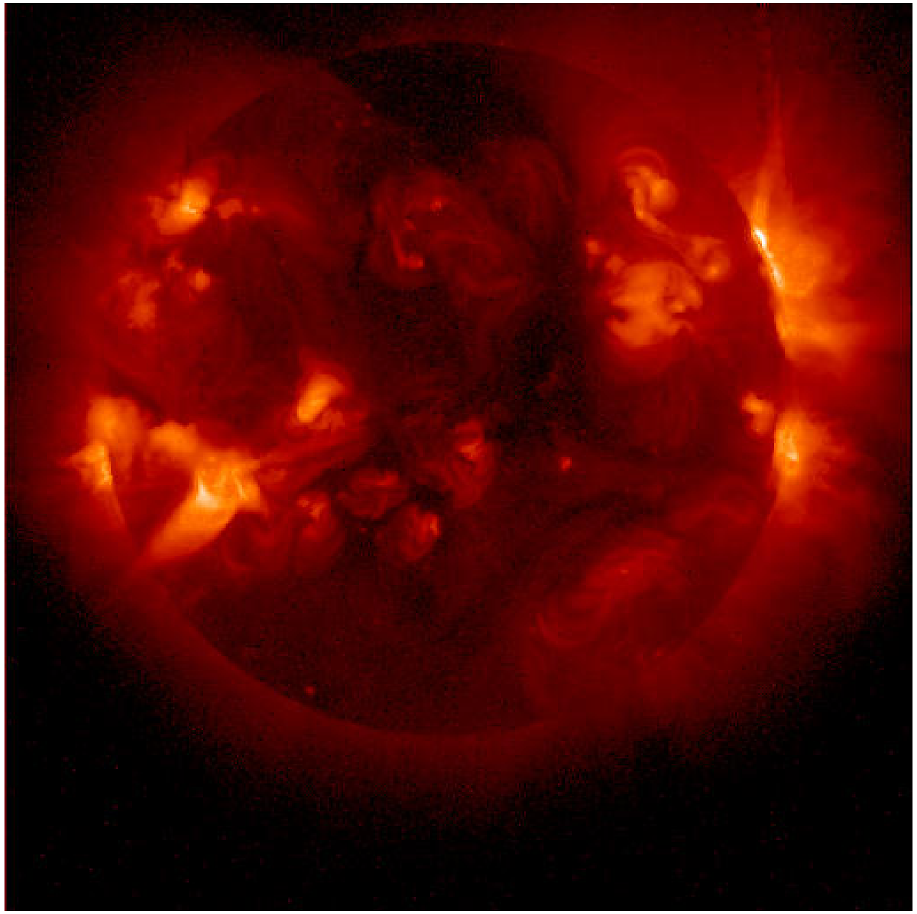
Structures in the corona:

– Dim coronal holes. Fine structure: plumes

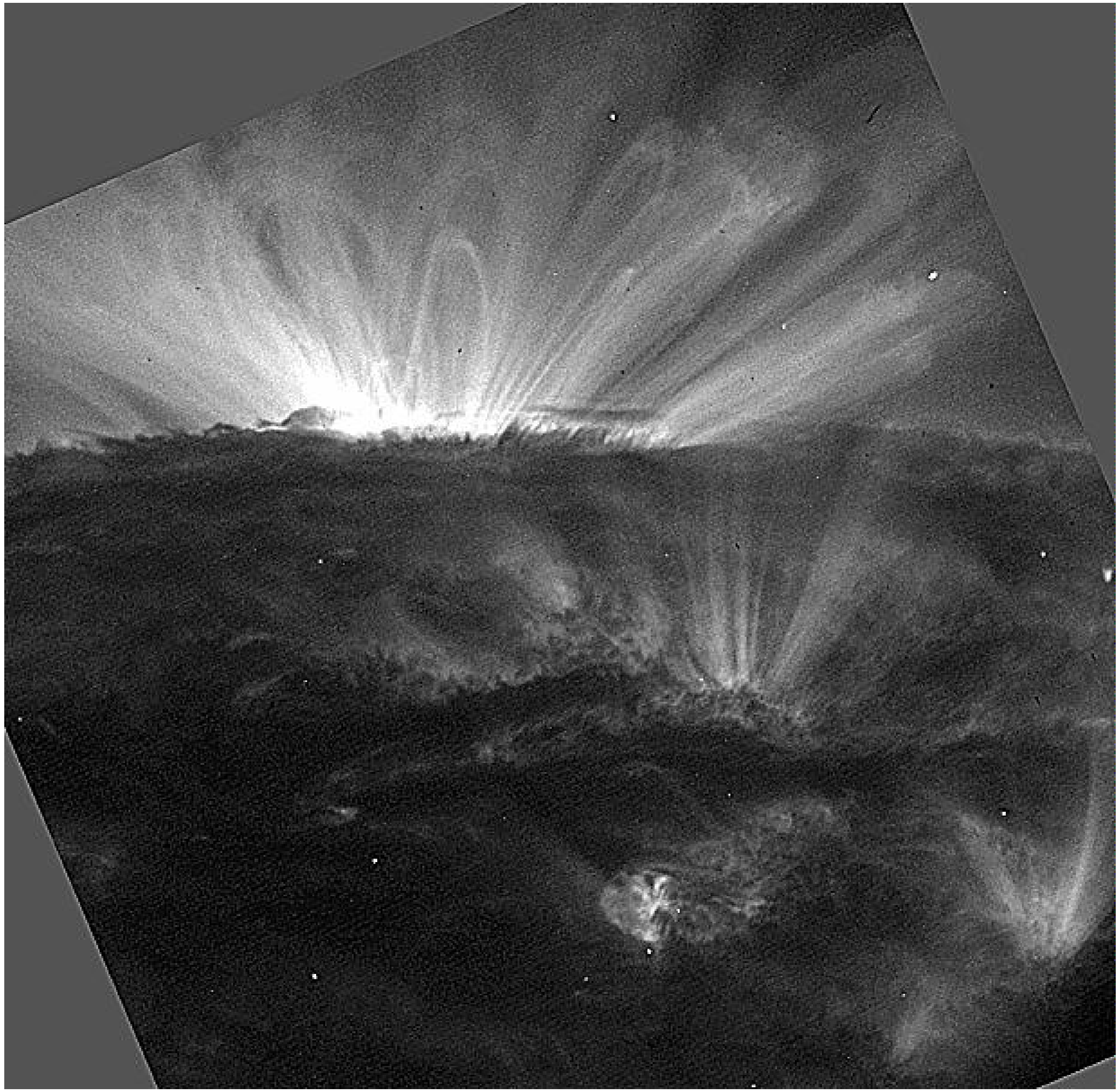
– Bright coronal loops over active regions.

– Streamers on top of loops.

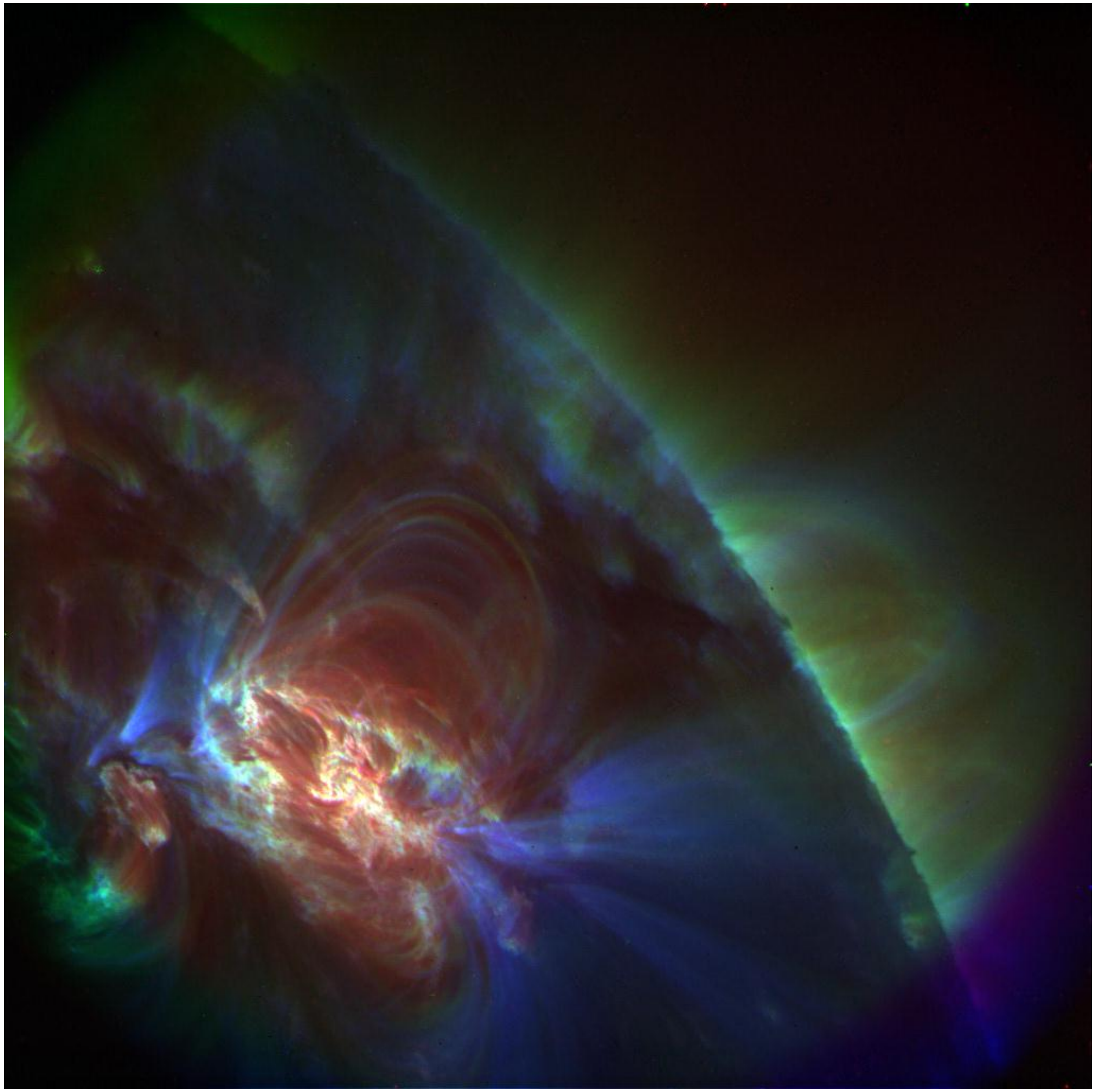




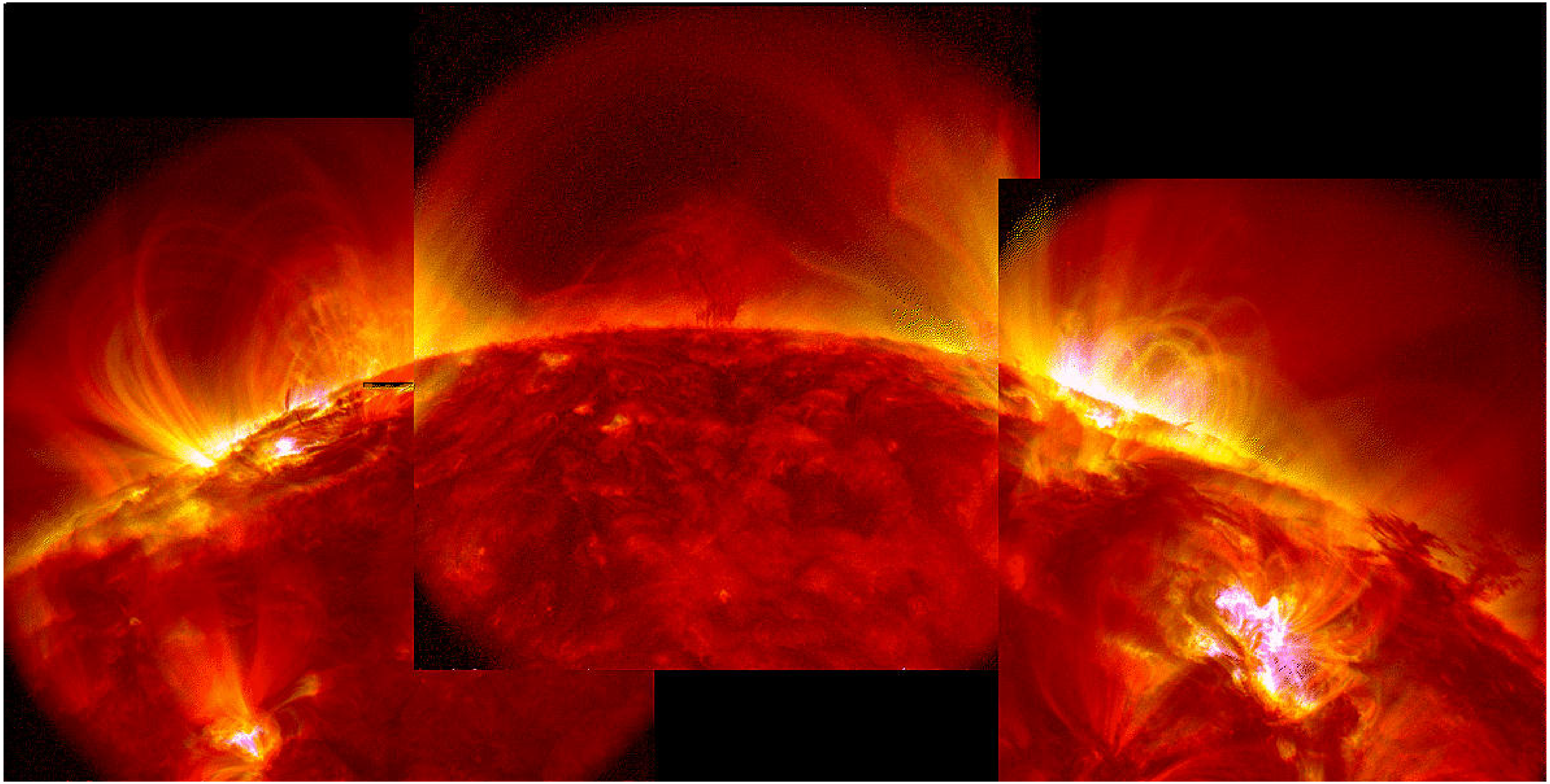
Yohkoh SXT



TRACE Fe IX



TRACE EUV composite



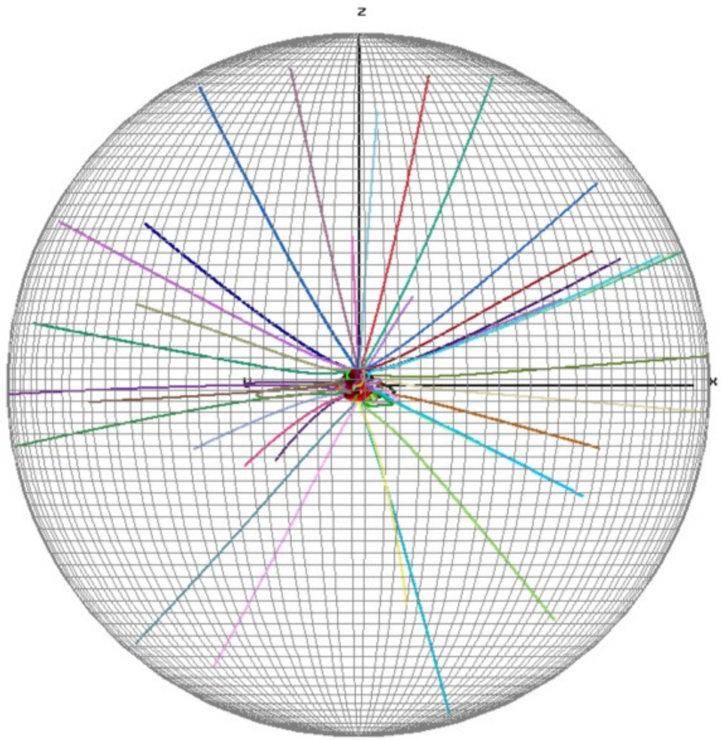
TRACE

FORCE-FREE MAGNETIC FIELDS

Plasma beta in tokamaks normally $\beta < 0.04$; in solar corona even lower $\Rightarrow$

$B^2/2\mu_0 \gg P \gg \rho v^2$ $\Rightarrow$ all other forces negligible compared to mg.forces.

Force-free field: $\mathbf{j} \times \mathbf{B} \propto (\nabla \times \mathbf{B}) \times \mathbf{B} = 0 \quad \Rightarrow \quad \boxed{\nabla \times \mathbf{B} = \alpha_{\mathrm{ff}} \mathbf{B}}$

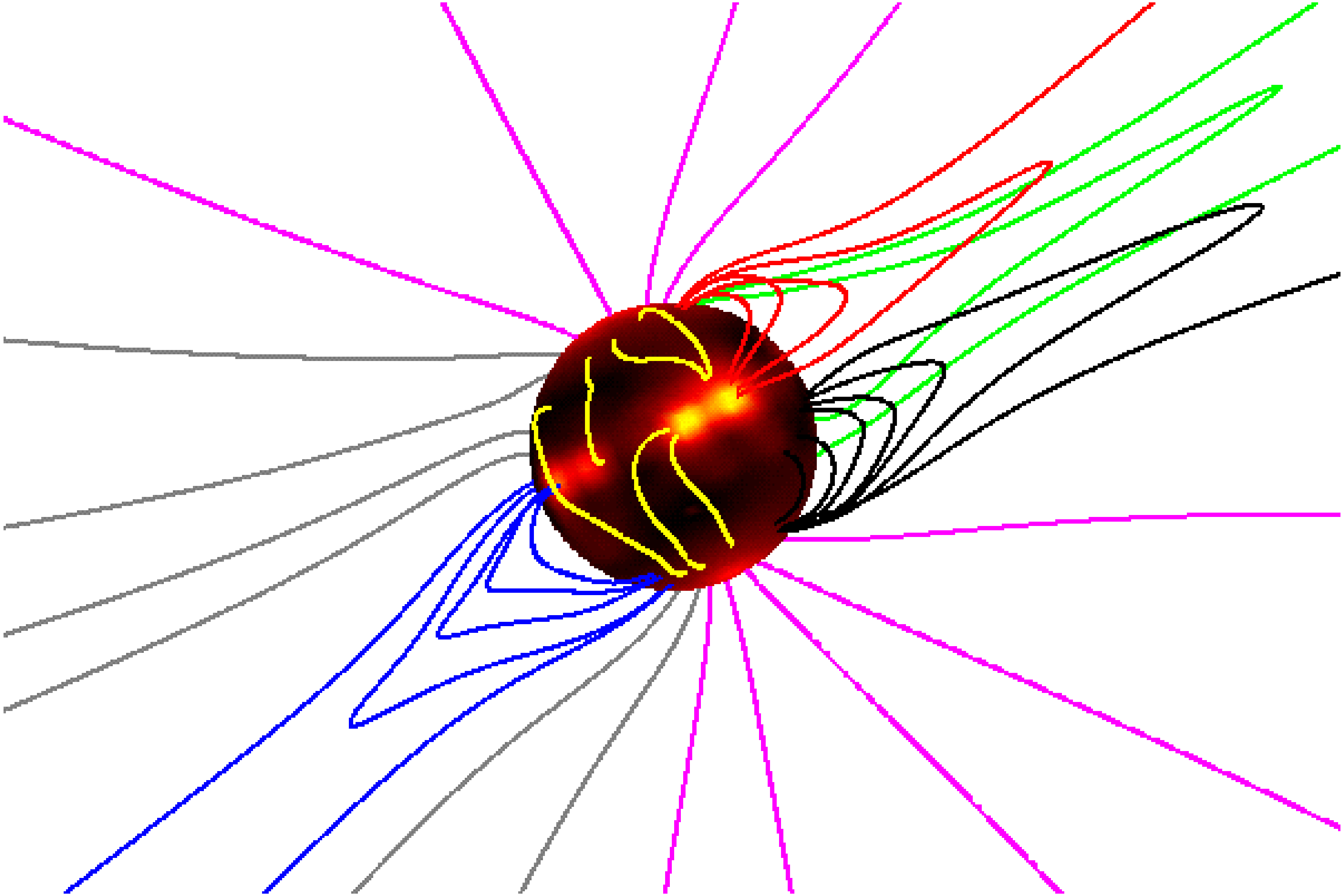


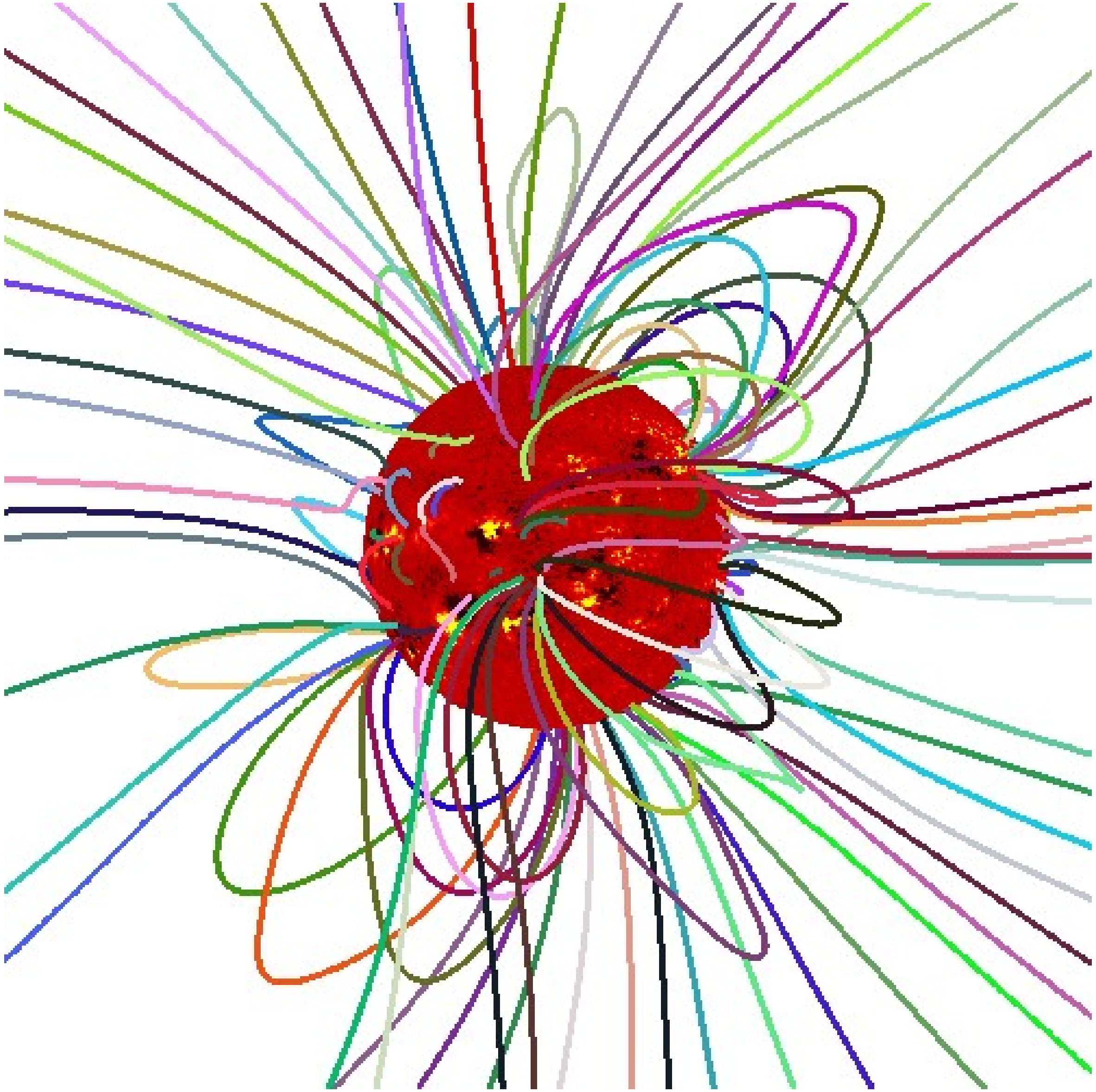
Potential field: $\alpha_{\mathrm{ff}} \equiv 0$

Corresponds to minimum energy configuration,
for a given photospheric magnetogram.

Actual calculation: source surface method.

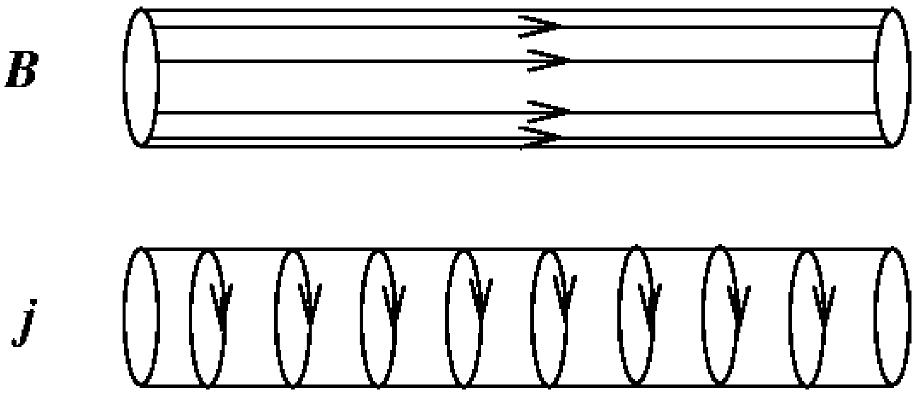
B assumed to be radial at $R_{\mathrm{out}} \gg R_\odot$:



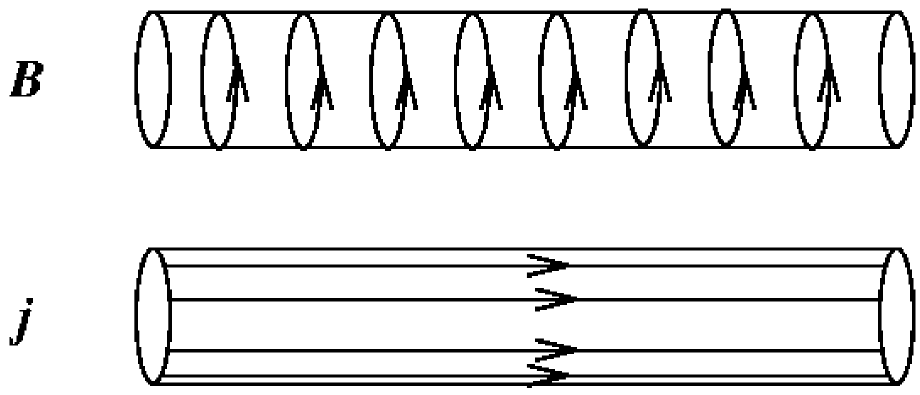


NON-POTENTIAL FIELDS AND HELICITY

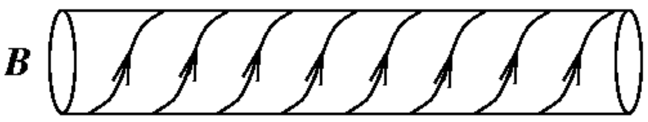
Straight flux tube:

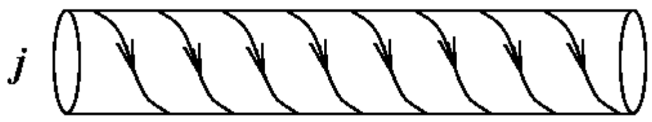


Straight current tube:



Combining them ≡ twisting them:





$\Rightarrow$ A measure of twist is the *helicity:*

current helicity:	$h_C = \mathbf{B} \cdot (\nabla \times \mathbf{B})$.
magnetic helicity:	$h_M = \mathbf{A} \cdot \mathbf{B} \equiv \mathbf{A} \cdot (\nabla \times \mathbf{A})$
kinetic helicity:	$h_K = \mathbf{v} \cdot (\nabla \times \mathbf{v})$
cross helicity:	$h_X = \mathbf{v} \cdot \mathbf{B}$

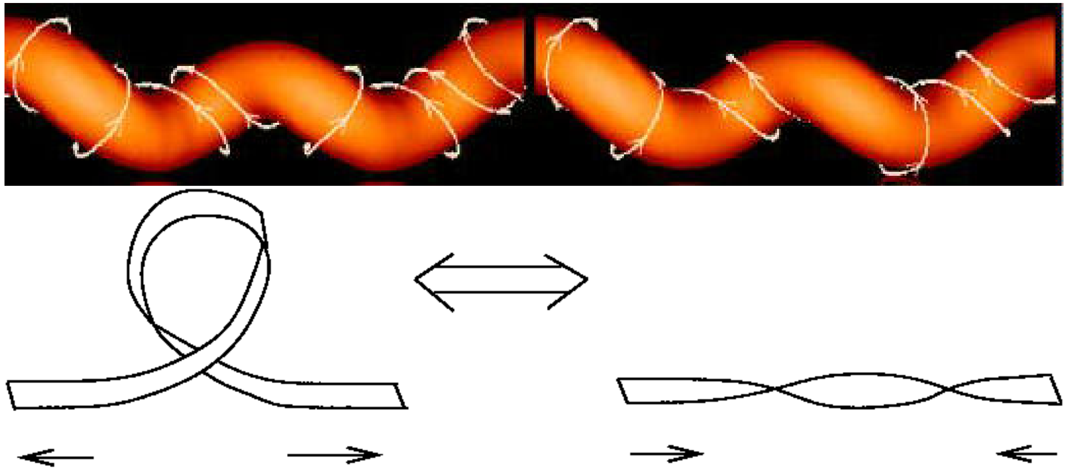
(NB these are helicity densities. Total helicity: $H = \int h\, dV$)

Relation of H_M, H_C, and α_{ff}:

– $\alpha_{ff} = H_C / B^2$
– No simple relation for H_M and H_C; but are generally related monotonically.
– Observationally, we determine $\alpha_{ff} \Rightarrow H_C$ and from this infer H_M.

OTHER CONTRIBUTIONS TO HELICITY

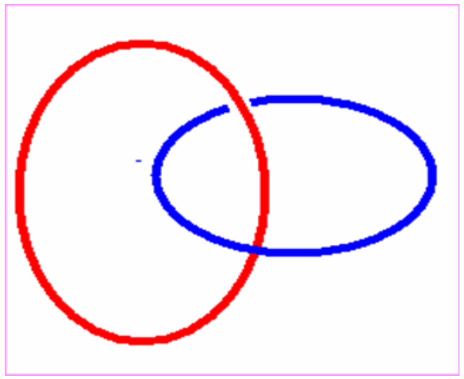
Writhe (or kink)



(Kink is writhe that's been "ironed out".)

Kink and twist can be converted
into each other:

Interlinkage



Splitting a closed flux tube with
one full twist/perimeter, this is what you get:

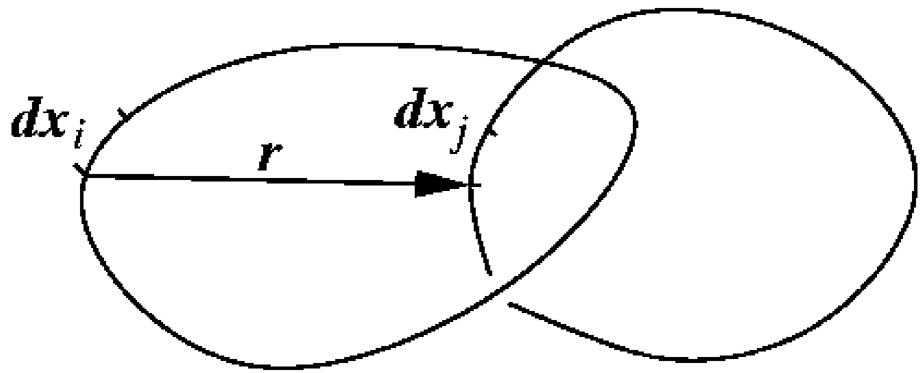
Twist + Writhe = "Self-helicity"

Interlinkage = "Mutual helicity" (M. Berger's terminology)

HELICITY AND TOPOLOGY

Gauss *linking number* for two closed curves (asteroid orbits, electric circuits...):

Half of the minimum number of crossings on a plane projection



$$L_{ij} = \frac{1}{4\pi} \oint \oint \frac{\mathbf{r}}{r^3} \, d\mathbf{x}_i \times d\mathbf{x}_j$$

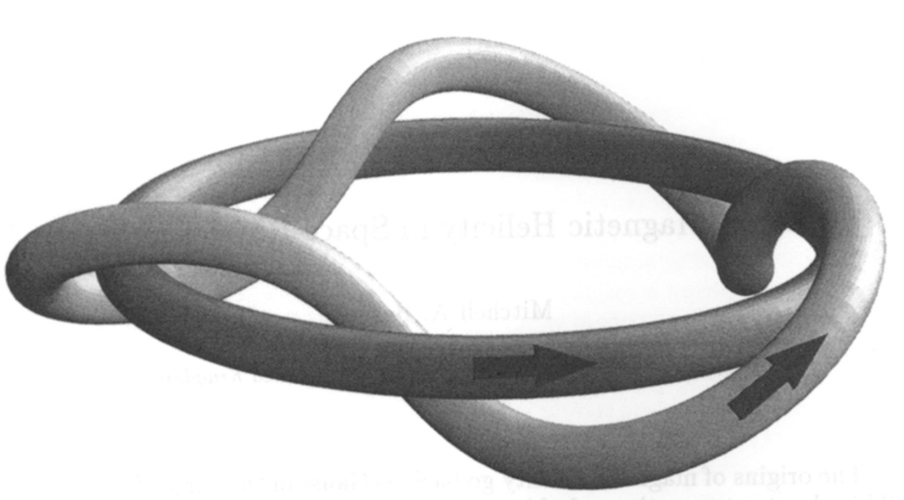
Consider a closed system of field lines ($B_n = 0$ on boundary). Divide it into $N \to \infty$ infinitesimal flux tubes.

In Coulomb gauge, $\mathbf{A} = -\frac{1}{4\pi} \int \frac{\mathbf{r}}{r^3} \times \mathbf{B}(\mathbf{x}) \, d^3\mathbf{x}$.

$$\Rightarrow \; H_M = -\frac{1}{4\pi} \int \int \mathbf{B}(\mathbf{y}) \frac{\mathbf{r}}{r^3} \times \mathbf{B}(\mathbf{x}) \, d^3\mathbf{x} \, d^3\mathbf{y} = \sum_i \sum_j L_{ij} \Phi_i \Phi_j$$

$\Rightarrow$ Magnetic helicity is the sum (integral) of the linking number over each pair of infinitesimal flux tubes!

E.g. $L = 3$:



For a twisted, closed flux tube, then

$$H_M = L\Phi^2$$

where L is the linking number of any fibril around the axis, or another fibril.

Problem: What is the magnetic helicity of a "Möbius flux tube"?
What happens if we split it along its axis?

CONSERVATION OF MAGNETIC HELICITY

Woltjer (1958):

In ideal MHD ($\sigma = \infty$), in a closed volume ($B_n = v_n = 0$), H_M =const.

$\Rightarrow$ Magnetic helicity is an ideal MHD invariant.

(Also if not closed but $\partial_t \mathbf{A} = 0$ on boundary.)
Allow for some resistivity. Resistive timescale is L^2/η.

Dissipation of energy can be enhanced by turbulence and reconnection.

But these do not enhance dissipation of H_M!

In summary:

– Energy:	direct cascade,	effective dissipation at small scales
– Magnetic helicity:	inverse cascade,	ineffective dissipation

HELICITY BALANCE IN OPEN DOMAINS:

FORCE-FREE CONFIGURATIONS

Line-tying boundary conditions: fix $\mathbf{A}_h$ on boundary: $\delta\mathbf{A}\times\mathbf{n}=0$

(≡ fix B_n and field line connections: $\delta\mathbf{A}\times\mathbf{n}=0$)

Question: what is the minimum energy configuration? Theorems:

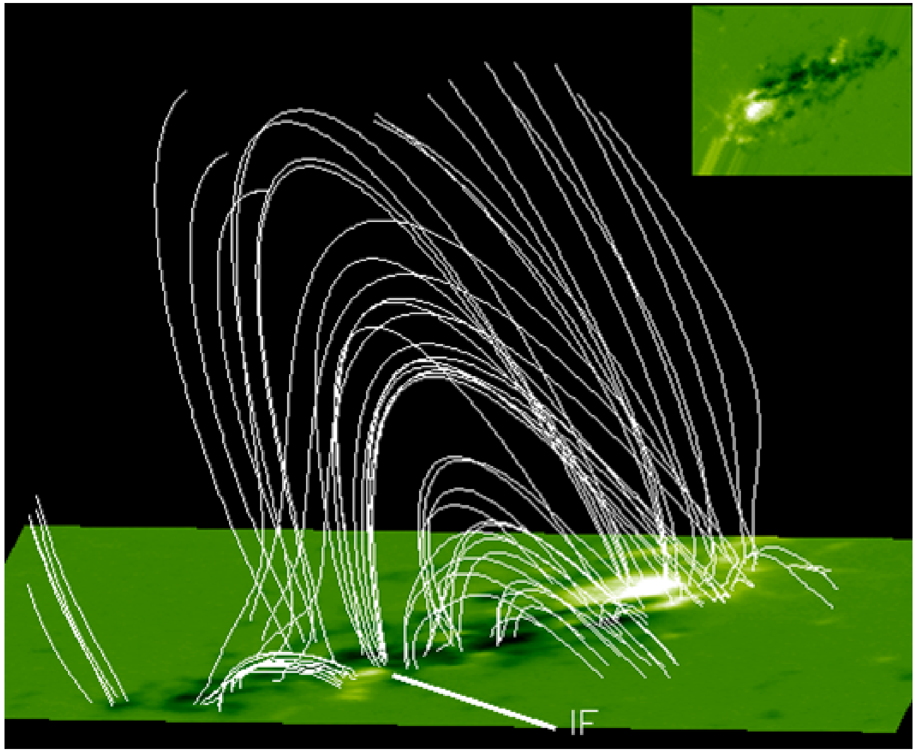
(1) Minimum energy theorem:

For ideal MHD perturbations the force-free condition

$$\boxed{\nabla\times\mathbf{B}=\alpha_{ff}\mathbf{B}}$$

is necessary for energy extremum ($\delta E=0$).

(2) Woltjer's theorem (1958): for H_M-conserving perturbations ($\delta H_M=0$): constant-α_{ff} linear force-free field is a necessary and sufficient condition.



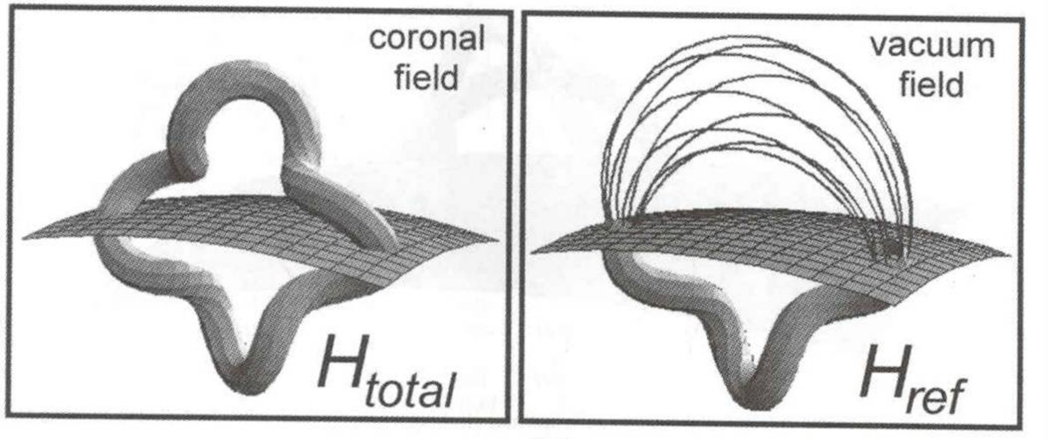
Conjectures:

(3) Woltjer–Taylor conjecture:

The constant-α_{ff} state can actually be reached on short timescale by Taylor relaxation (many small reconnections).

(4) The potential field is an absolute energy minimum.

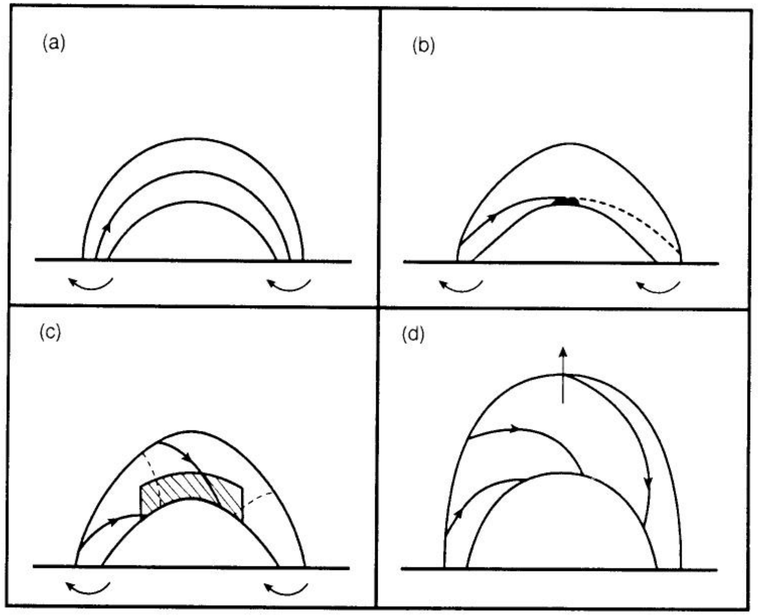
It can only be reached by getting rid of helicity.



(5) Aly–Sturrock conjecture:

for a given photospheric footpoint distribution,
open mg.field has maximal energy.

The sources of coronal activity: twisted flux loops

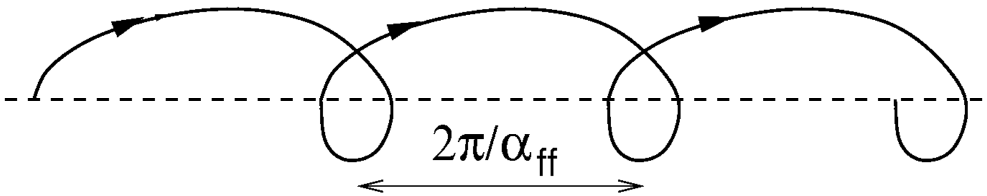


Flux ropes emerging into the atmosphere are twisted.

In depressions of the twisted structure plasma can be pooled ⇒ quiescent prominences.

An increase in twist may lead to eruption ⇒ eruptive prominences, CMEs.

A measure of twist (helicity): $\alpha_{\rm ff}$

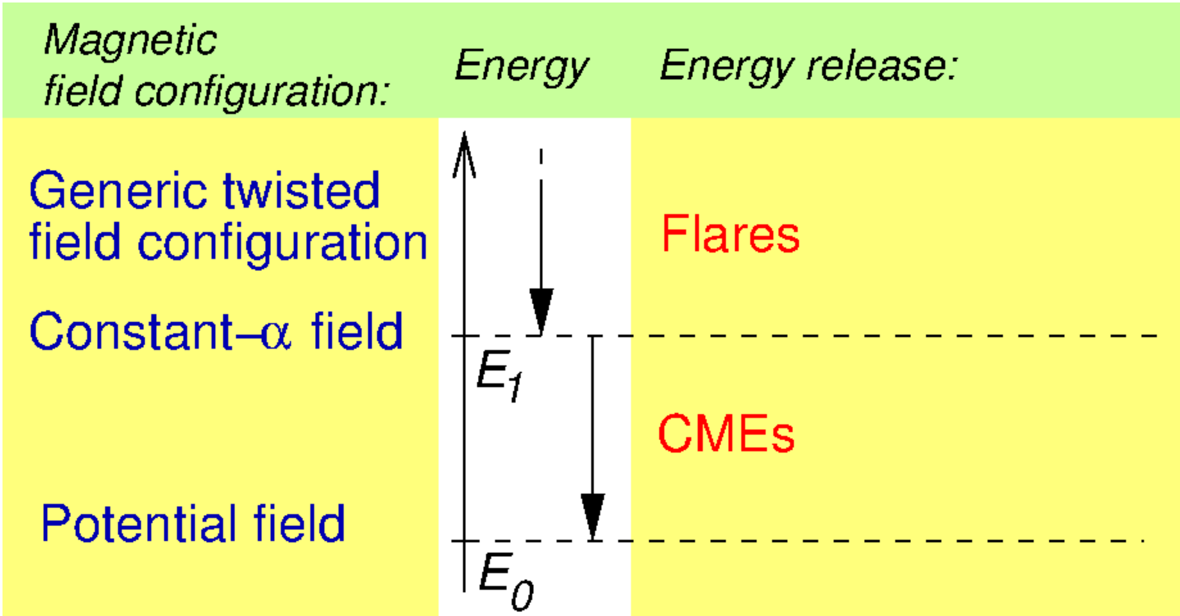


In a force free field, $\alpha_{\rm ff}$ is constant along field lines.

For given photospheric boundary conditions

minimal energy (E_0) field: potential field — $\alpha_{\rm ff} \equiv 0$

minimal energy field (E_1) for a given total helicity: $\alpha_{\rm ff} \equiv$ const.



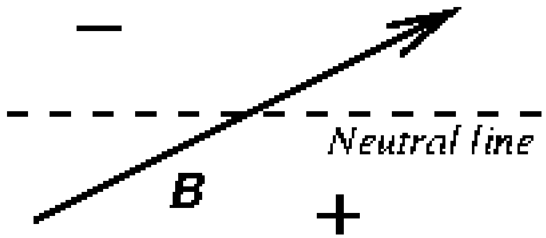
Free magnetic energy: $E_M - E_0 \Rightarrow$ activity phenomena.

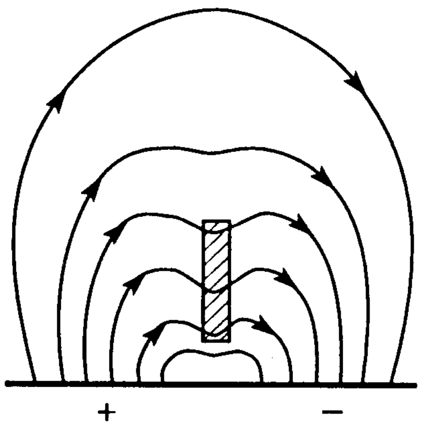
flares (reconnection): energy released ($E_M - E_1$), but total helicity is conserved.

CMEs $\Rightarrow$ Sun gets rid of helicity and $E_1 - E_0$.

Magnetic types and models of filaments:

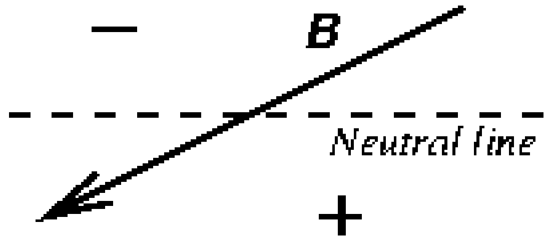
Normal (N–type):

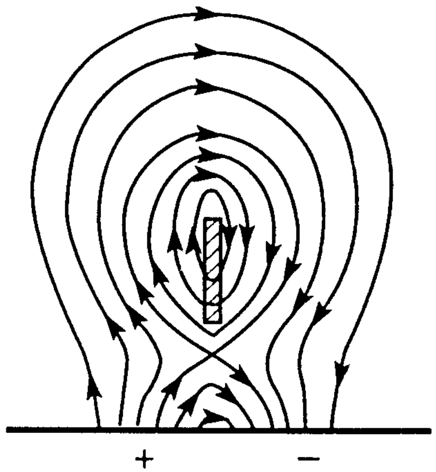




Kippenhahn-Schlüter model (1957)

Inverse (I–type):

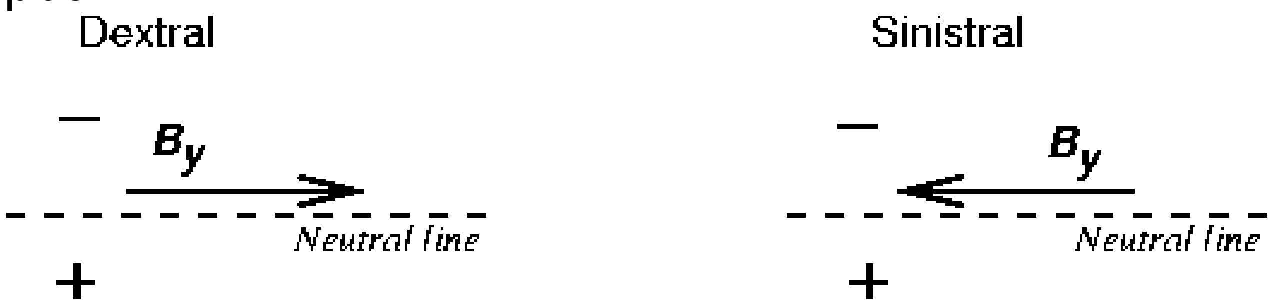




Kuperus-Raadu model (1974)

I-type more common, esp. among polar filaments. N-type filaments all below 30 Mm

Chirality types:



Dextral dominates on N hemisphere, sinistral in S.

Formation of filaments

Quiescent filaments may form

- (A) In active regions [less common]
- (B) Between two AR
- (C) Over polar crown neutral line, near expanding AR

Most accepted model: condensation in/above sheared arcade, aided by siphon flow.

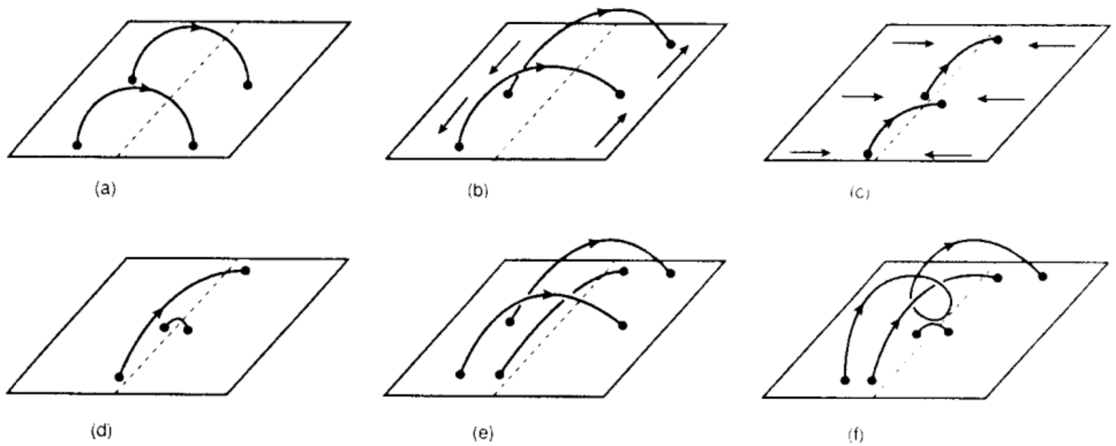
(Pikel'ner 1971, Choe & Lee 1992)

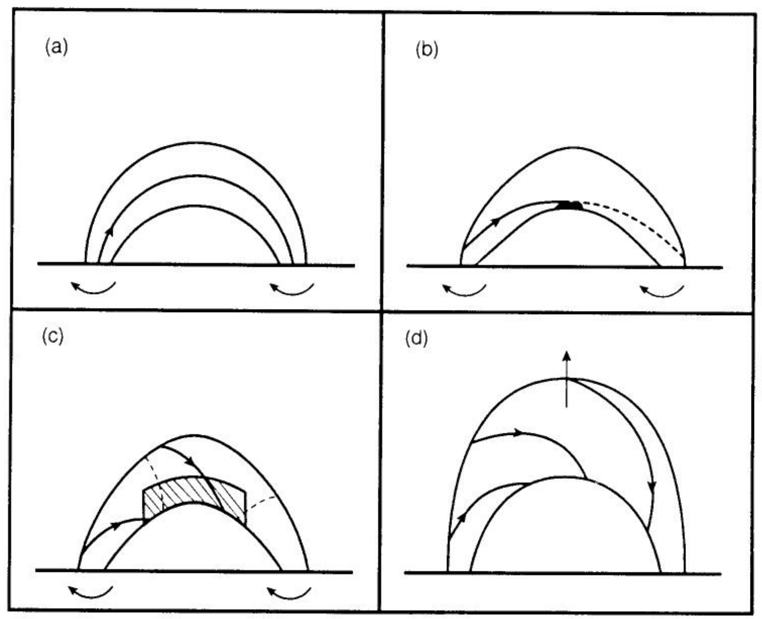
Moderate shear: N-type filament possible. Very high shear: N-type equilibrium disappears (Hood & Anzer 1990)

Reconnection of strongly sheared arcade ⇒ I-type prominence lying in helical flux tube. (Pneuman 1983)

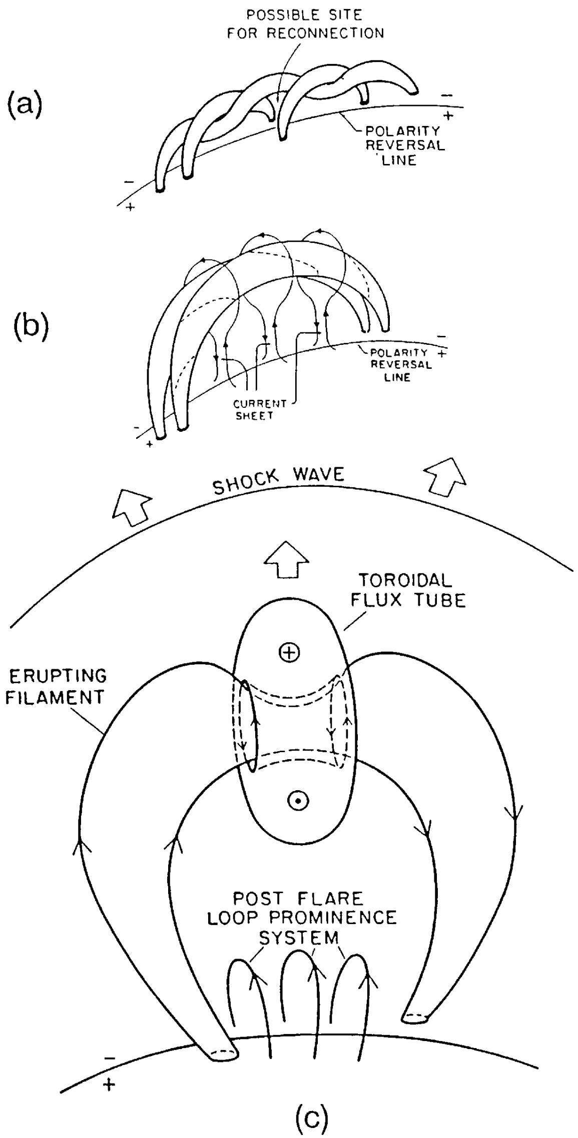
Problem: shear needed for chirality rule opposite to diff.rot.

Alternative: twisted flux tube





SOLAR ERUPTIONS



"Solar eruption" (aka eruptive solar flare) = flare + CME + eruptive prominence.

How does the arcade let the prominence go?

Observations suggest "tether cutting"
(Sturrock 1989)

But: Aly–Sturrock conjecture:
for a given photospheric footpoint distribution, open (radial) mg.field has maximal energy.

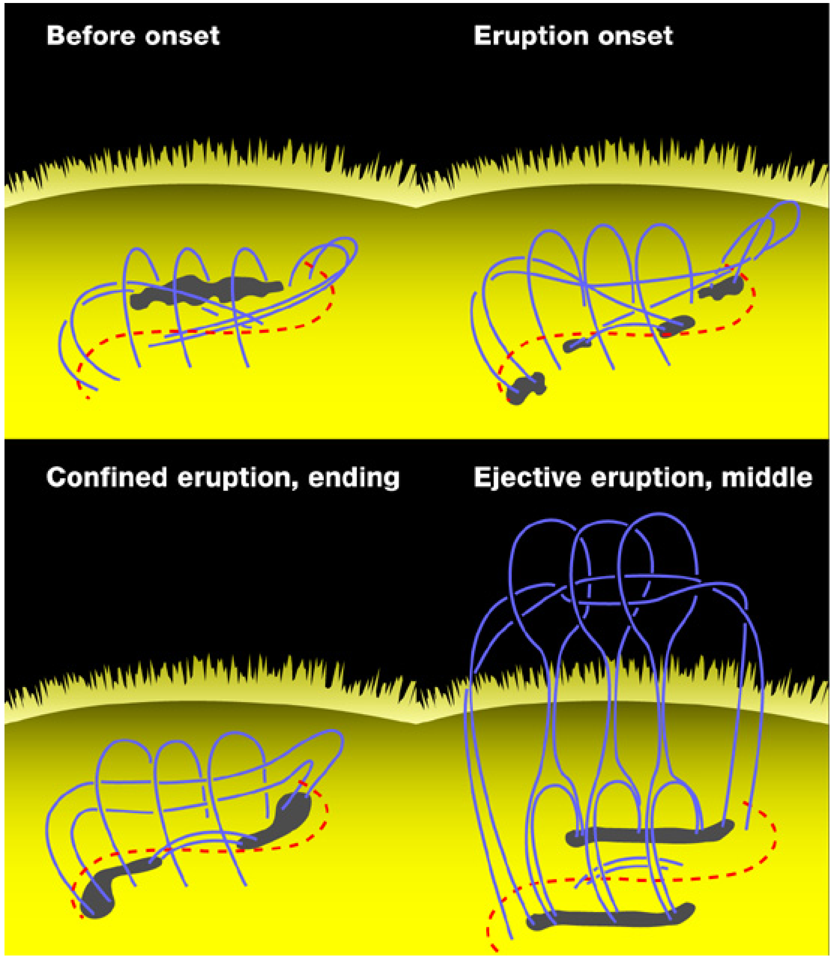
(Only holds for dipolar footpoint arangements.)

Possible mechanisms:

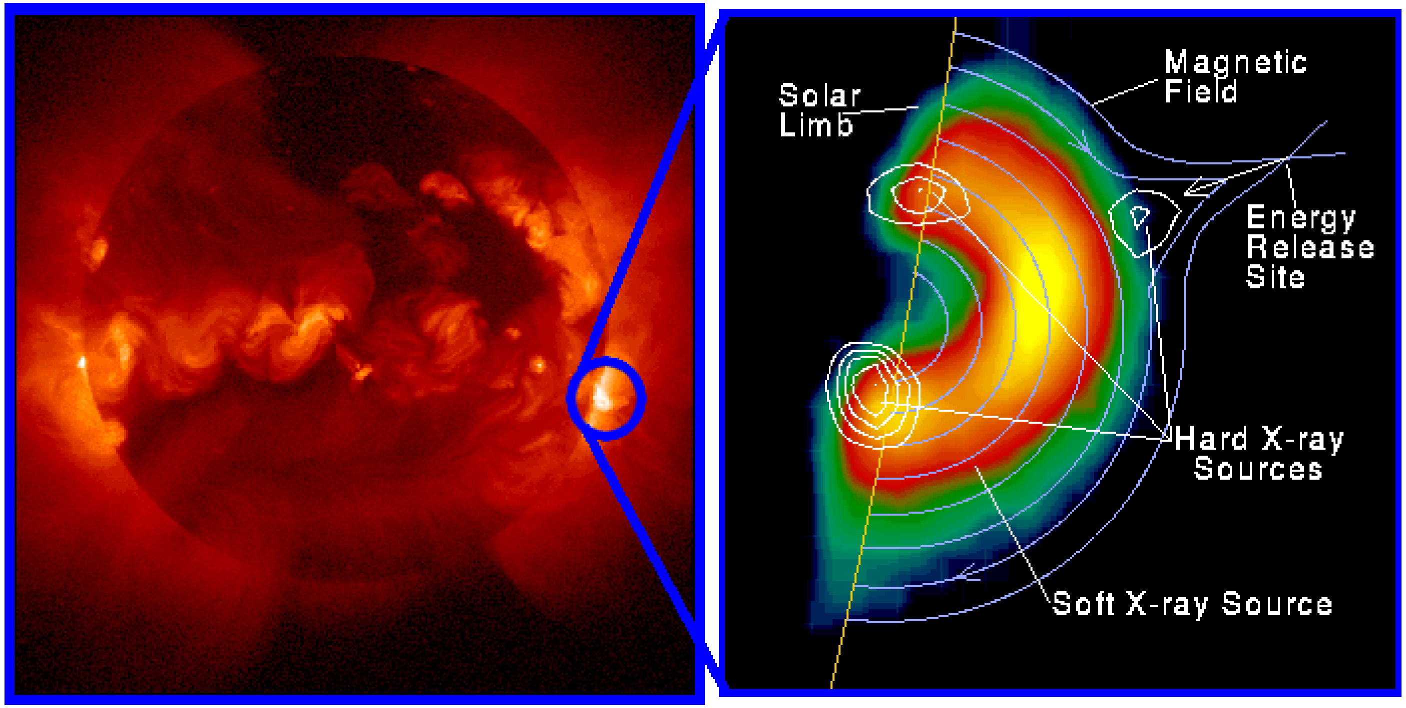
– Flux cancellation (Low 1977; Amari et al. 2000): Footpoints converge, then cancel
$\Rightarrow$ prominence rises, then erupts.
Footpoint change $\Rightarrow$ Aly-Sturrock invalid.

– Breakout (Antiochos 1998): Quadrupolar config. $\Rightarrow$ Aly–Sturrock invalid.
Arcade reconnects with overlying field and opens up.

– Kink instability (Török & Kliem 2004): Twisted loop gets more twisted
$\Rightarrow$ rises, then erupts. Arcade opens up in 3D.

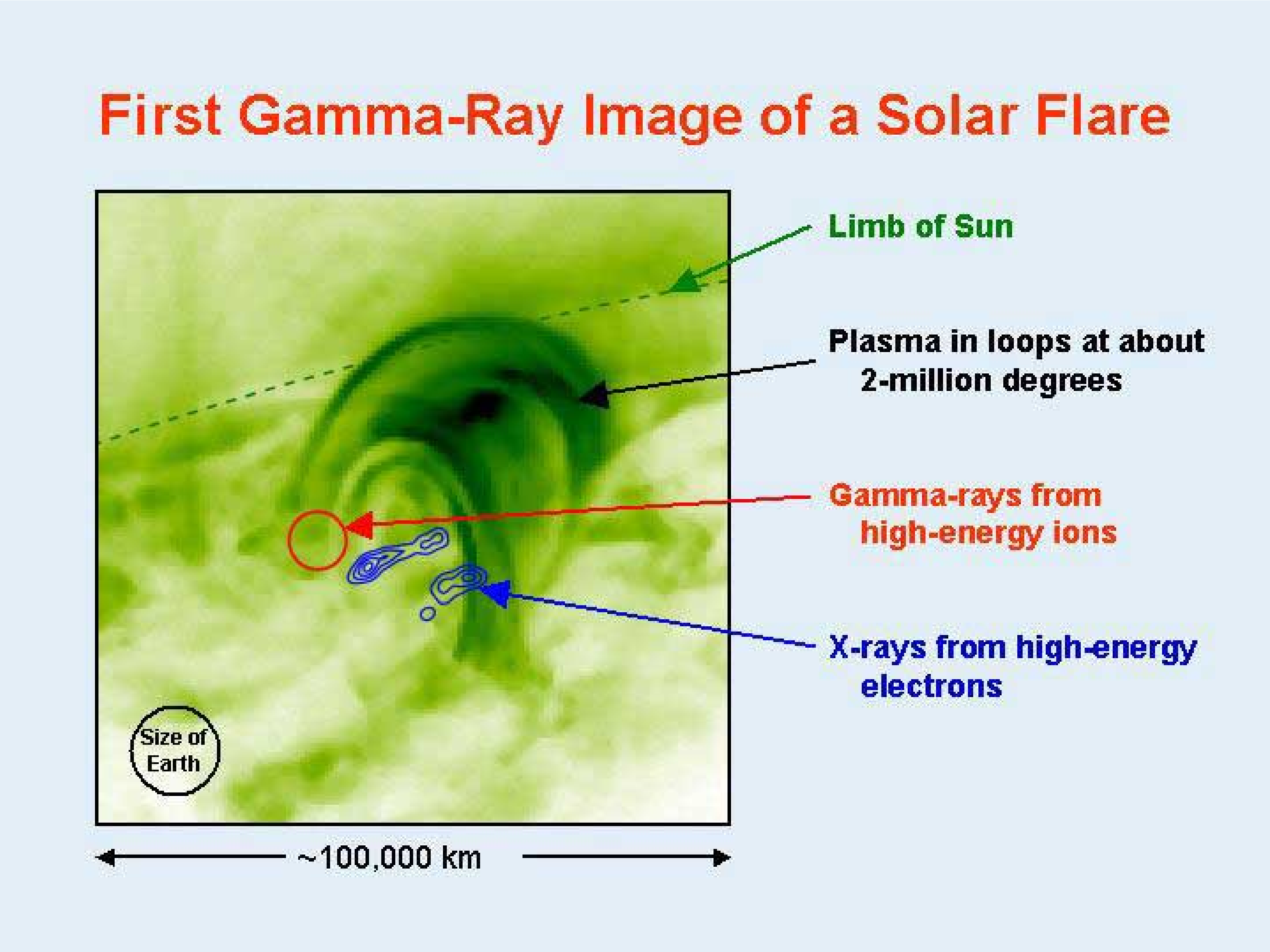

Before onset
Eruption onset
Confined eruption, ending
Ejective eruption, middle

Postflare loops:



Yohkoh X-ray Image of a Solar Flare, Combined Image in Soft X-rays (left) and Soft X-rays with Hard X-ray Contours (right). Jan 13, 1992.

Old postflare loops cool, but new, hot loops keep forming above them
$\Rightarrow$ flare involves change in magnetic structure: magnetic reconnection.


First Gamma-Ray Image of a Solar Flare
Limb of Sun
Plasma in loops at about 2-million degrees
Gamma-rays from high-energy ions
X-rays from high-energy electrons
Size of Earth
~100,000 km

Particle radiation in flares

Proton flares: Flares accompanied by > 10 MeV proton shower.

(Up to ~ 10 GeV. "Solar cosmic rays": > 1 GeV)

Shower arrives in 10-20 minutes, lasts 10-20 minutes

PCA — Polar Cap Absorption. Ionospheric increase at high latitudes

Caused by 1–600 MeV electrons. They arrive in 1–2 hours.

Geomagnetic activity related to flares: magnetic storms, aurorae.

Due to 1-100 keV electrons. They arrive in 1–2 days.

$\Rightarrow$ Thick target model: Particles emitted downwards cause observed flare emission