Topological protection of quantum information

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Kvantummérés Lendület csoport

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- Protection of Quantum Information?
  - Quantum Error Correction
- Topological?
  - Majorana Wire
  - Experiments

1. **Quantum Error Correction**

2. Topological by nature: Majorana Wire

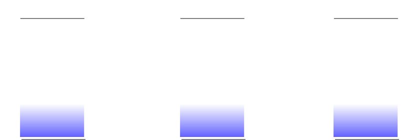
Classical: Dissipation discretizes errors. Majority voting corrects them

1 bit, 2 states in RAM:
voltage > 0.5V, voltage < 0.5V

- Small errors (charge leakage, voltage creep) → correct by charge refresh, dissipation

- Big error = bit flip: correct using redundancy

1) use 3 physical bits
For 1 logical bit



2) If not equal (syndrome),
flip the minority bit (correct).

Quantum information is fragile

$$|\Psi\rangle = a\,|0\rangle + b\,|1\rangle\,; \quad a, b \in \mathbb{C}$$

- One qubit, 2 real parameters (normalization, global phase unimportant)→ Continuous information, small errors
- Cannot use dissipation to discretize
- Measurement can destroy information

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \qquad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix};$$

Redundancy protects against single bit flip errors

$$|0\rangle \to |\bar{0}\rangle = |000\rangle$$
$$|1\rangle \to |\bar{1}\rangle = |111\rangle$$

$$a|0\rangle + b|1\rangle \to a|000\rangle + b|111\rangle$$

Bit flip, X takes us out of computational space.

$$a|010\rangle + b|101\rangle$$

1) Syndrome Measurement:

$$(Z_2 \oplus Z_3, Z_1 \oplus Z_3) = (1, 0)$$

⊕ is XOR = + (mod 2)
Nonlocal measurement → Reveals position of error without measuring the value

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix};$$

Problem: phase errors stay in computational subspace

$$|\Psi\rangle = a\,|000\rangle + b\,|111\rangle$$

$$Z_1\,|\Psi\rangle = Z_2\,|\Psi\rangle = Z_3\,|\Psi\rangle = a\,|000\rangle - b\,|111\rangle$$

Redundancy just increases error probability.

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix};$$

Extra redundancy in different basis protects against phase errors too

$$|0\rangle \rightarrow |\bar{0}\rangle = (|000\rangle + |111\rangle)\,(|000\rangle + |111\rangle)\,(|000\rangle + |111\rangle)$$

$$|1\rangle \rightarrow |\bar{1}\rangle = (|000\rangle - |111\rangle)\,(|000\rangle - |111\rangle)\,(|000\rangle - |111\rangle)$$

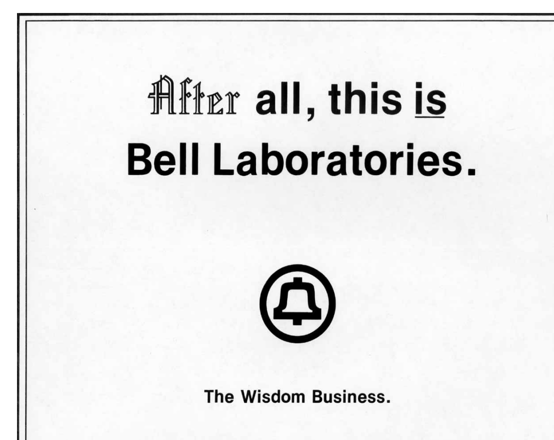
1) Syndrome Measurement:

$$(X_1 X_2 X_3 X_4 X_5 X_6,\ \ X_4 X_5 X_6 X_7 X_8 X_9)$$

Shor code, 1995.

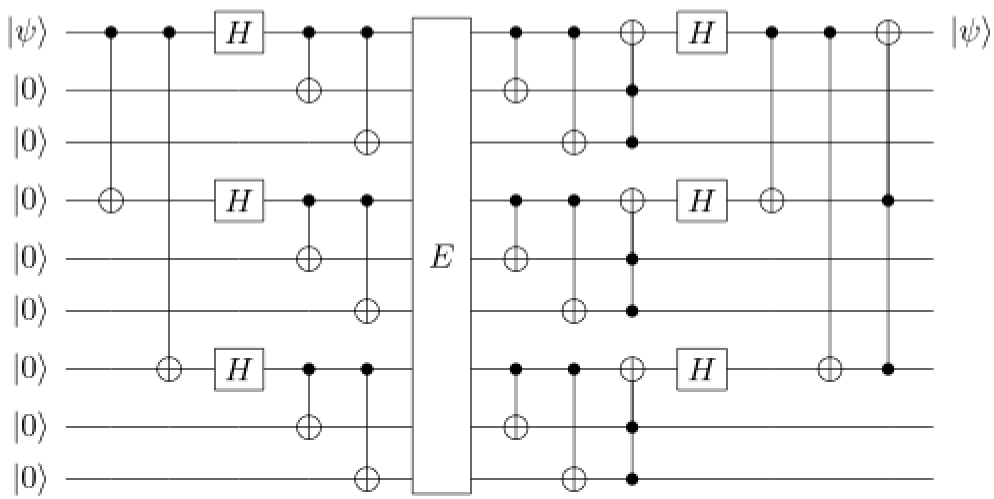
Peter Shor

- *1959
- MIT / Berkeley / Bell Laboratories
- 1994: Quantum Factoring Algorithm
- 1995: First Quantum Error Correcting Code
- MIT Applied Mathematics
- 2002, King Faisal Prize for Science (200 k$)



Shor code: computational states are highly entangled

$$|0\rangle \rightarrow |\bar{0}\rangle = (|000\rangle + |111\rangle)\,(|000\rangle + |111\rangle)\,(|000\rangle + |111\rangle)$$
$$|1\rangle \rightarrow |\bar{1}\rangle = (|000\rangle - |111\rangle)\,(|000\rangle - |111\rangle)\,(|000\rangle - |111\rangle)$$



Entanglement protects information against local errors

- Local errors, e.g., decoherence: even small errors leave computational subspace
  - → can be diagnosed by syndrome measurement,
  - → can be corrected
- Logical operations (quantum gates) have to be very entangled

Error correction possible if gates are very precise (error threshold 1%)

- Error correction: number of extra gates should scale polynomially
- Error threshold depends on scheme,
  10^{-5} … 10^{-2} error probability

- What is the best architecture for error correction?

- Can we use error-proof quantum hardware?

Literature

- John Preskill lecture Quantum Information lecture notes

1. Quantum Error Correction

2. **Topological by nature: Majorana Wire**

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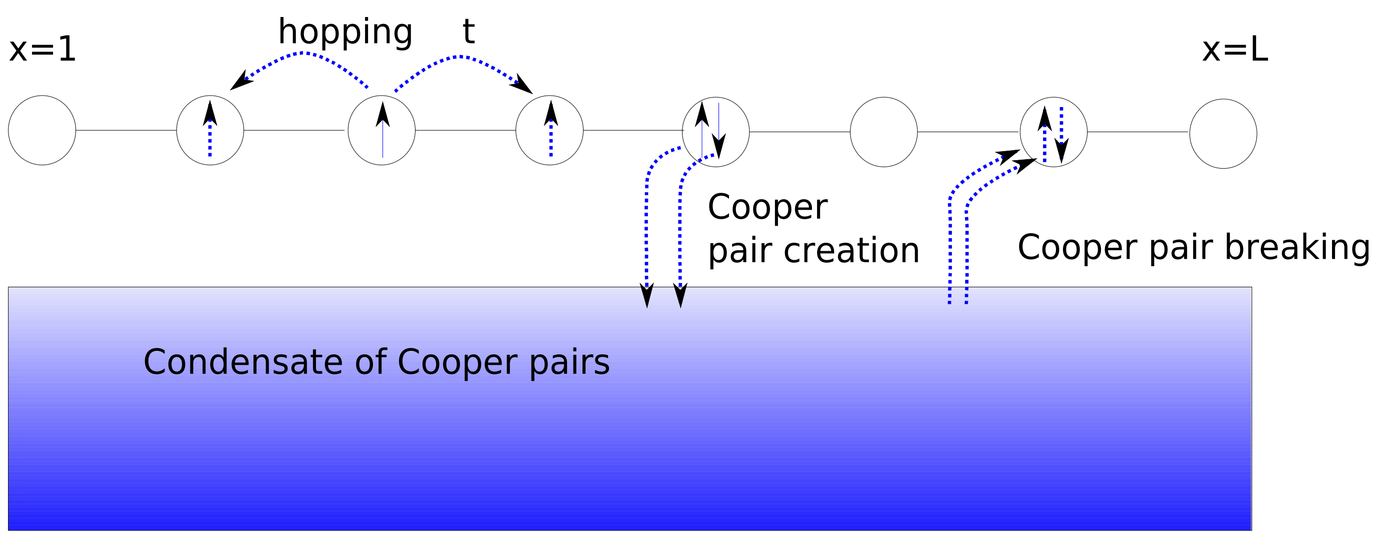
- Can we use error-proof quantum hardware?

Alexey Kitaev

- *1963
- Landau Institute / Microsoft Research
- 2001: Kitaev Wire
- 2006: Toric Code
- CalTech, Theoretical Physics & Maths
- 2012: Fundamental Physics Prize (3 M$ = 2x Nobel)

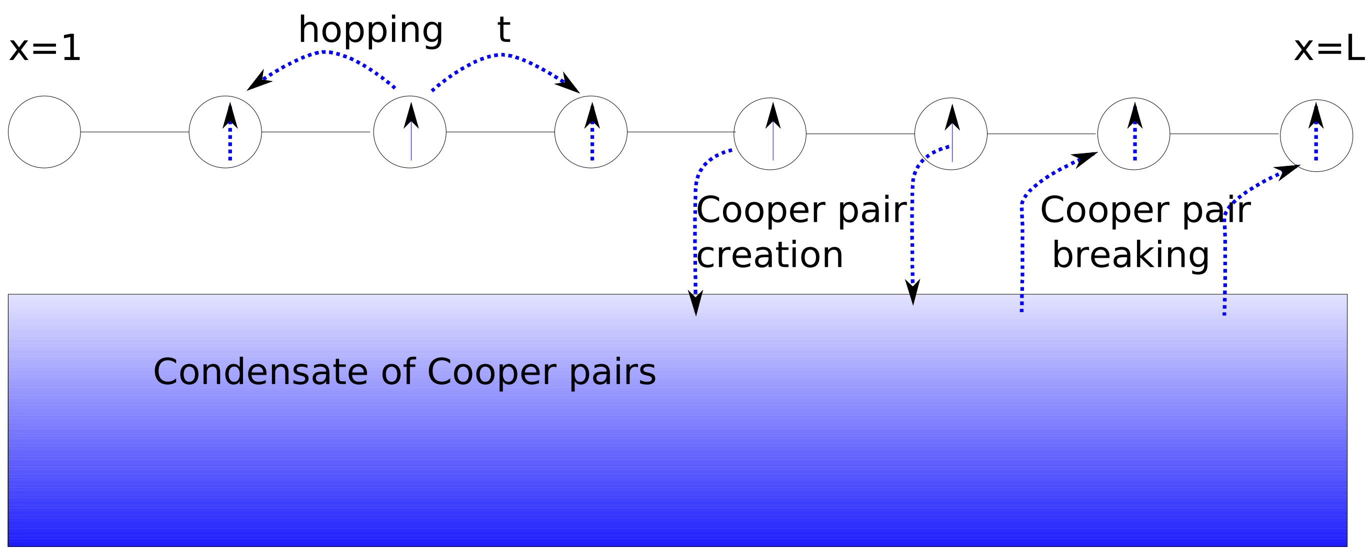
Microsoft

Minimal model for superconductors: Cooper pairs created/broken at all sites



$$\hat{H} = -\mu \sum_{x=1}^{L} \sum_{s=\uparrow,\downarrow} \hat{c}^{\dagger}_{x,s}\hat{c}_{x,s} - t \sum_{x=1}^{L} \sum_{s=\uparrow,\downarrow} \hat{c}^{\dagger}_{x,s}\hat{c}_{x+1,s} + h.c. - |\Delta|\, e^{i\phi} \sum_{x=1}^{L} \hat{c}_{x,\uparrow}\hat{c}_{x,\downarrow} + h.c.$$

Kitaev's minimal model for spinless, *p-wave* superconductor



$$H = -\mu \sum_{x=1}^{L} \hat{c}_x^{\dagger} \hat{c}_x - t \sum_{x=1}^{L} \hat{c}_x^{\dagger} \hat{c}_{x+1} + h.c. - |\Delta| \, e^{i\phi} \sum_{x=1}^{L} \hat{c}_x \hat{c}_{x+1} + h.c.$$

Single spin component → drop spin index

[2001, Kitaev, Physics Uspekhi]

Quadratic Hamiltonian → can be seen as noninteracting, free particles

$$\hat{H} = -\mu \sum_{x=1}^{L} \hat{c}_x^\dagger \hat{c}_x - t \sum_{x=1}^{L} \hat{c}_x^\dagger \hat{c}_{x+1} + h.c. - |\Delta|\, e^{i\phi} \sum_{x=1}^{L} \hat{c}_x \hat{c}_{x+1} + h.c.$$

$$\hat{H} = \sum_{l=1}^{L} E_l \hat{d}_l^\dagger \hat{d}_l$$

Noninteracting eigenstates: linear combinations of electrons and holes:

$$\hat{d}_l = \sum_x u_{lx} \hat{c}_x + v_{lx} \hat{c}_x^\dagger$$

Fermions: $\{\hat{d}_l, \hat{d}_m\} = 0$ $\{\hat{d}_l, \hat{d}_m^\dagger\} = \delta_{lm}$

Can be restricted to positive energy $E_l > 0$

The coefficients u_{lx} and v_{lx} are the components of the wavefunction of γ_l

$$\hat{H} = -\mu \sum_{x=1}^{L} \hat{c}_x^\dagger \hat{c}_x - t \sum_{x=1}^{L} \hat{c}_x^\dagger \hat{c}_{x+1} + h.c. - |\Delta| \, e^{i\phi} \sum_{x=1}^{L} \hat{c}_x \hat{c}_{x+1} + h.c.$$

$$\hat{H} = \sum_{l=1}^{L} E_l \hat{d}_l^\dagger \hat{d}_l$$

Noninteracting eigenstates: linear combinations of electrons and holes:

$$\hat{d}_l = \sum_x u_{lx} \hat{c}_x + v_{lx} \hat{c}_x^\dagger$$

Fermionic anticommutation relations → Normalized, orthogonal wavefunctions

Ground state has no excitations.

$$|GS\rangle = \prod \hat{d}_l \,|0\rangle$$

Superconductor ground state, complicated

Empty state containing no electrons, simple

The wavefunctions of the excitations are found via the Bogoliubov-de Gennes trick

1. Rewrite the Hamiltonian

$$\hat{H} = \sum_{l=1}^{L} E_l \hat{d}_l^\dagger \hat{d}_l = \frac{1}{2}\sum_{l=1}^{L} E_l \hat{d}_l^\dagger \hat{d}_l - \frac{1}{2}\sum_{l=1}^{L} E_l \hat{d}_l \hat{d}_l^\dagger + \frac{1}{2}\sum_{l} E_l$$

2. Use shorthand

$$c^\dagger = (\hat{c}^\dagger_{1,\uparrow}, \hat{c}^\dagger_{1,\downarrow}, \ldots, \hat{c}^\dagger_{N,\uparrow}, \hat{c}^\dagger_{N,\downarrow});$$
$$c = (\hat{c}_{1,\uparrow}, \hat{c}_{1,\downarrow}, \ldots, \hat{c}_{N,\uparrow}, \hat{c}_{N,\downarrow});$$

$$\hat{H} = \sum_{\alpha,\beta} c^\dagger_\alpha h_{\alpha,\beta} c_\beta + \frac{1}{2} c^\dagger_\alpha \Delta_{\alpha,\beta} c^\dagger_\beta + + \frac{1}{2} c_\beta \Delta^*_{\alpha,\beta} c_\alpha;$$

$$\hat{H} = \frac{1}{2}\begin{pmatrix} c^\dagger & c \end{pmatrix} \begin{pmatrix} h & \Delta \\ -\Delta^* & -\tilde{h} \end{pmatrix} \begin{pmatrix} c \\ c^\dagger \end{pmatrix} + \frac{1}{2}\mathrm{Tr} h.$$

Eigenvectors of H_{bdG} are the wavefunctions of quasiparticles

3. Introduce the single-particle Hamiltonian H_{bdG}

$$\hat{H} = \frac{1}{2}\begin{pmatrix} c^\dagger & c \end{pmatrix} \underbrace{\begin{pmatrix} h & \Delta \\ -\Delta^* & -\tilde{h} \end{pmatrix}}_{\mathcal{H}_{BdG}} \begin{pmatrix} c \\ c^\dagger \end{pmatrix} + \frac{1}{2}\mathrm{Tr}h.$$

j^{th} eigenvector of H_{bdG} is the (complex conjugate of) the wavefunction of γ_j.

4. Every free fermion d_j is represented twice – Particle-Hole Symmetry

Bogoliubov-de Gennes "trick" ensures Particle-Hole Symmetry

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} h & \Delta \\ -\Delta^* & -\tilde{h} \end{pmatrix}^* \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = - \begin{pmatrix} h & \Delta \\ -\Delta^* & -\tilde{h} \end{pmatrix}$$

$$\sigma_x K \mathcal{H}_{BdG} K \sigma_x = -\mathcal{H}_{BdG}$$

Exchange particles for holes

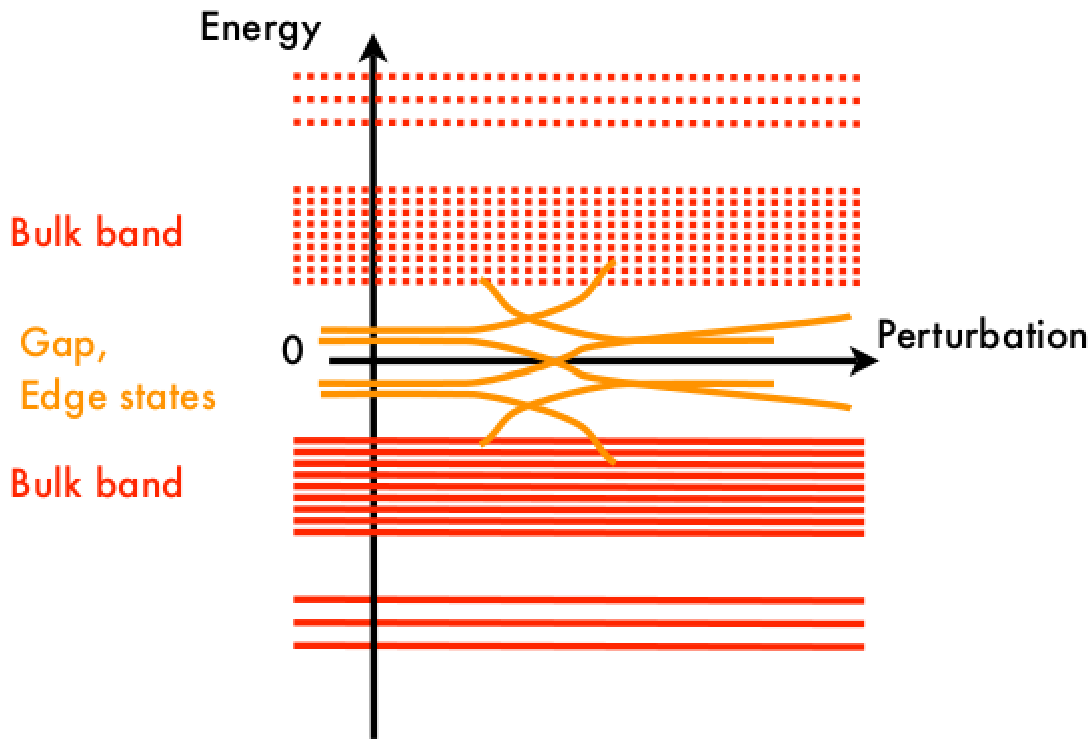
Complex conjugation in position space

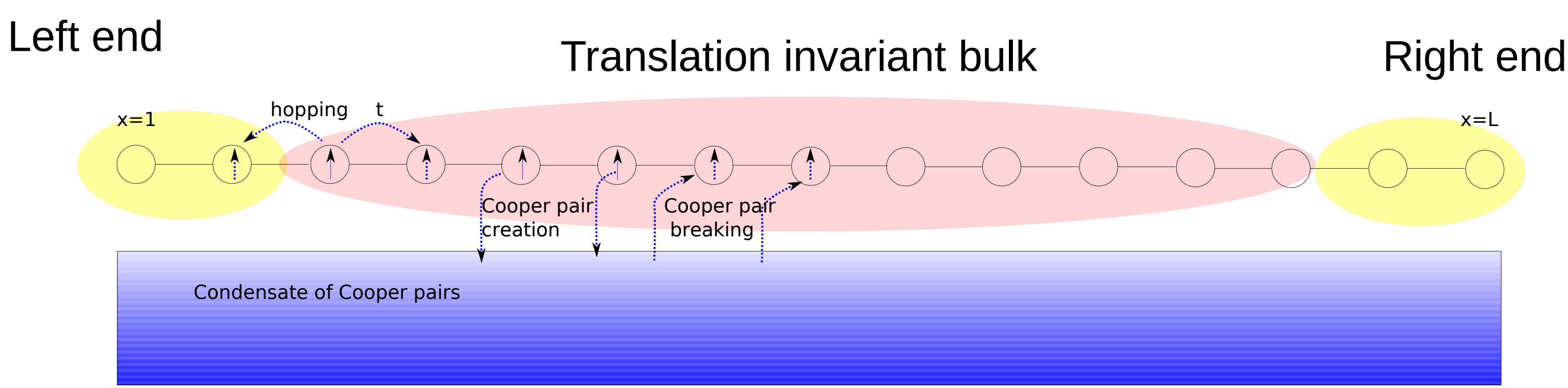
Every eigenstate of H_{bdG} has a particle-hole symmetric partner

$$\mathcal{H}_{BdG} |\Psi\rangle = E |\Psi\rangle$$

$$\mathcal{H}_{BdG} \sigma_x K |\Psi\rangle = -E \sigma_x K |\Psi\rangle$$

Particle-Hole Symmetry ensures Spectrum of H_{bdG} has to be symmetric





Majorana fermions: mathematical tool.

Decompose each fermion into “real and imaginary parts”

$$\hat{c}_x = \tfrac{1}{2} e^{-i\phi/2} (\hat{\gamma}_{B,x} + i \hat{\gamma}_{A,x}) \qquad \hat{\gamma}_{B,x} = e^{i\phi/2} \hat{c}_x + e^{-i\phi/2} \hat{c}_x^\dagger$$

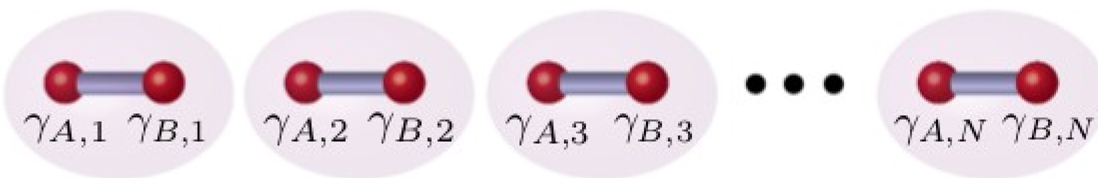
$$\hat{c}_x^\dagger = \tfrac{1}{2} e^{i\phi/2} (\hat{\gamma}_{B,x} - i \hat{\gamma}_{A,x}) \qquad \hat{\gamma}_{A,x} = -i \left(e^{i\phi/2} \hat{c}_x + i e^{-i\phi/2} \hat{c}_x^\dagger \right)$$

These Majorana operators are self-adjoint fermions

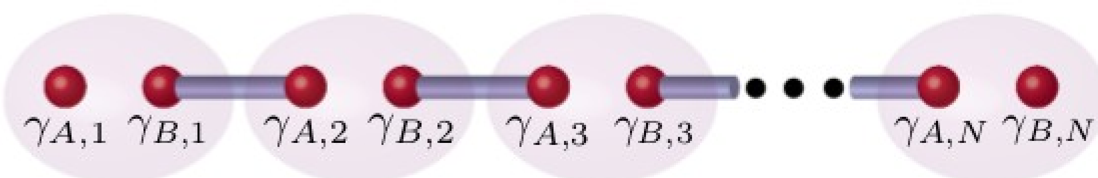
$$\{\hat{\gamma}_{A,x}, \hat{\gamma}_{B,x'}\} = 0; \quad \{\hat{\gamma}_{A,x}, \hat{\gamma}_{A,x'}\} = \{\hat{\gamma}_{B,x}, \hat{\gamma}_{B,x'}\} = 2\delta_{xx'}$$

Two simple limiting cases: disconnected sites vs equal hopping and pair potential

(a) $$t = \Delta = 0 \Longrightarrow \hat{H} = \frac{i}{2} \sum_x \hat{\gamma}_{A,x} \hat{\gamma}_{B,x}$$



(b) $$t = \Delta \neq \mu = 0$$



$$\hat{H} = \frac{it}{2} \sum_x \hat{\gamma}_{B,x} \hat{\gamma}_{A,x+1}$$

"topologically nontrivial"

The two Majorana fermions “left out” of the Hamiltonian form a zero-energy particle

$$\hat{d}_0 = \hat{\gamma}_{A,1} + i\hat{\gamma}_{B,N} \qquad \hat{H} = \sum_{l=1}^{N-1} E_l \hat{d}_l^\dagger \hat{d}_l$$

- Two Majoranas at the two ends combine to a zero-energy fermion
- Local excitations
- Equal weight particle and hole
- Their own particle-hole partners
- → Energy unchanged by local perturbations

Going away from the limiting case, the particle still has zero energy.

$$\hat{d}_0 = \hat{\gamma}_1 + i\hat{\gamma}_2$$

$$\hat{\gamma}_1 = a_1\hat{\gamma}_{A,1} + a_2\hat{\gamma}_{A,2} + \dots$$

$$\hat{\gamma}_2 = b_1\hat{\gamma}_{B,N} + b_2\hat{\gamma}_{B,N-1} + \dots$$

- In Bogoliubov-de Gennes picture, γ_1 and γ_2 are zero energy eigenstates
- Bulk gap → their wavefunctions remain exponentially localized
- They are their own particle-hole partners
- → Energy unchanged by local perturbations

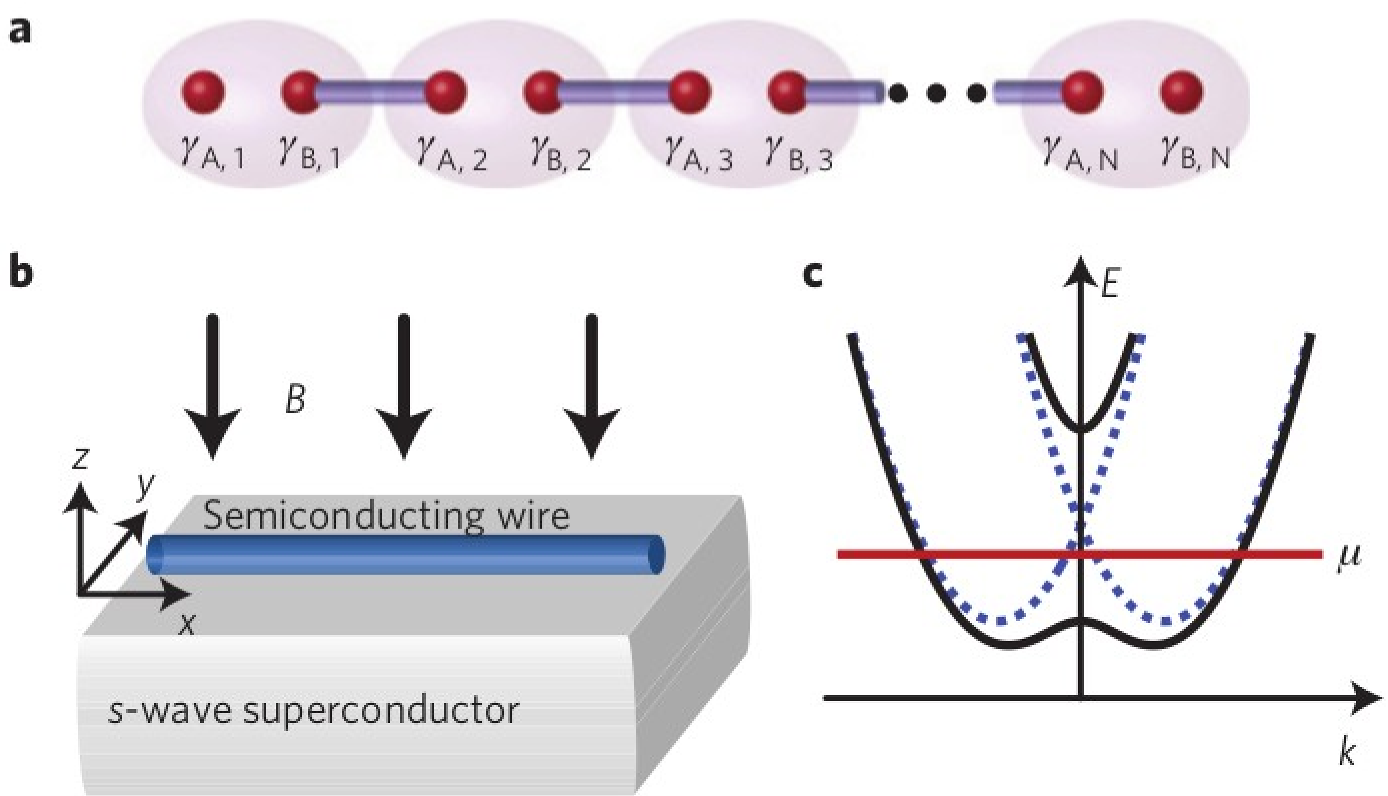
Quantum information can be hidden in Majorana modes

$$|\Psi\rangle = \alpha\,|G\rangle + \beta\hat{d}_0^{\dagger}\,|G\rangle$$

- Local environment cannot degrade the quantum information

→ no bit flips (d is a nonlocal particle)
→ no phase errors (d has zero energy)

Majoranas can be created in experiment



[Lutchyn et al, Oreg et al, PRL 2010]

Majoranas can be pushed around using a "keyboard" of electric bottom gates

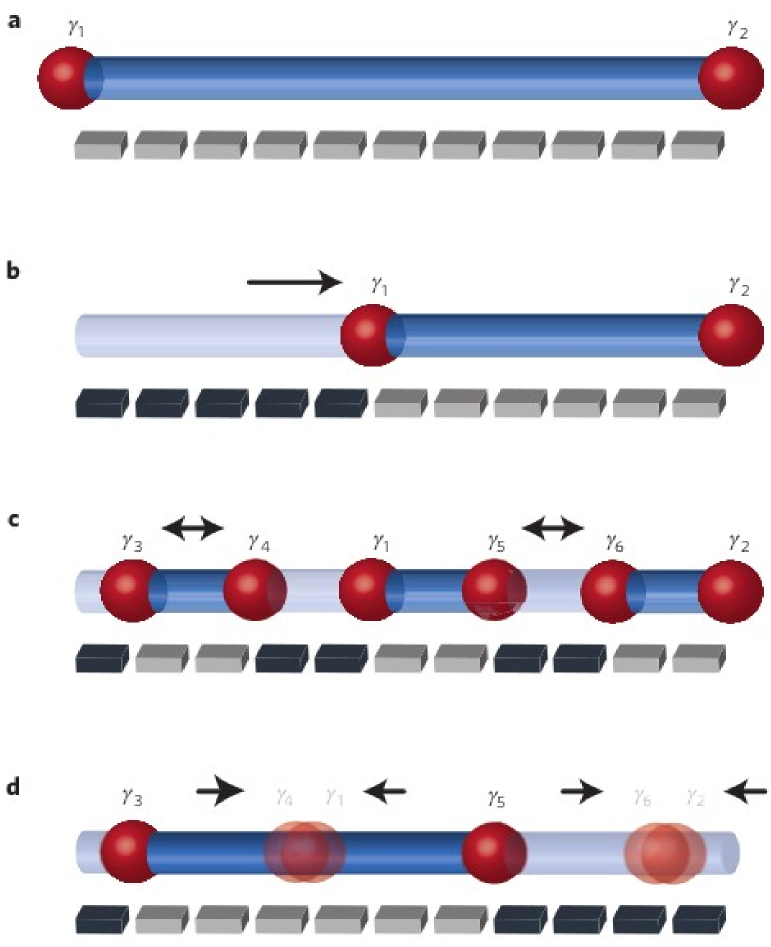


Figure 2 | Applying a 'keyboard' of individually tunable gates to the wire allows local control of which regions are topological (dark blue) and non-topological (light blue), and hence manipulate Majorana fermions while maintaining the bulk gap. As **a** and **b** illustrate, sequentially applying

- Local chemical potential μ controlled by voltage on bottom gates
- Move Majoranas
- Create or annihilate neighbouring Majoranas
- If change slow enough,

$$\tau \gg \frac{\hbar}{\delta H}$$

adiabatic limit: avoid exciting other modes

[Alicea et al, Nature Physics, 2011]

Some logical operations can be realized by braiding Majoranas

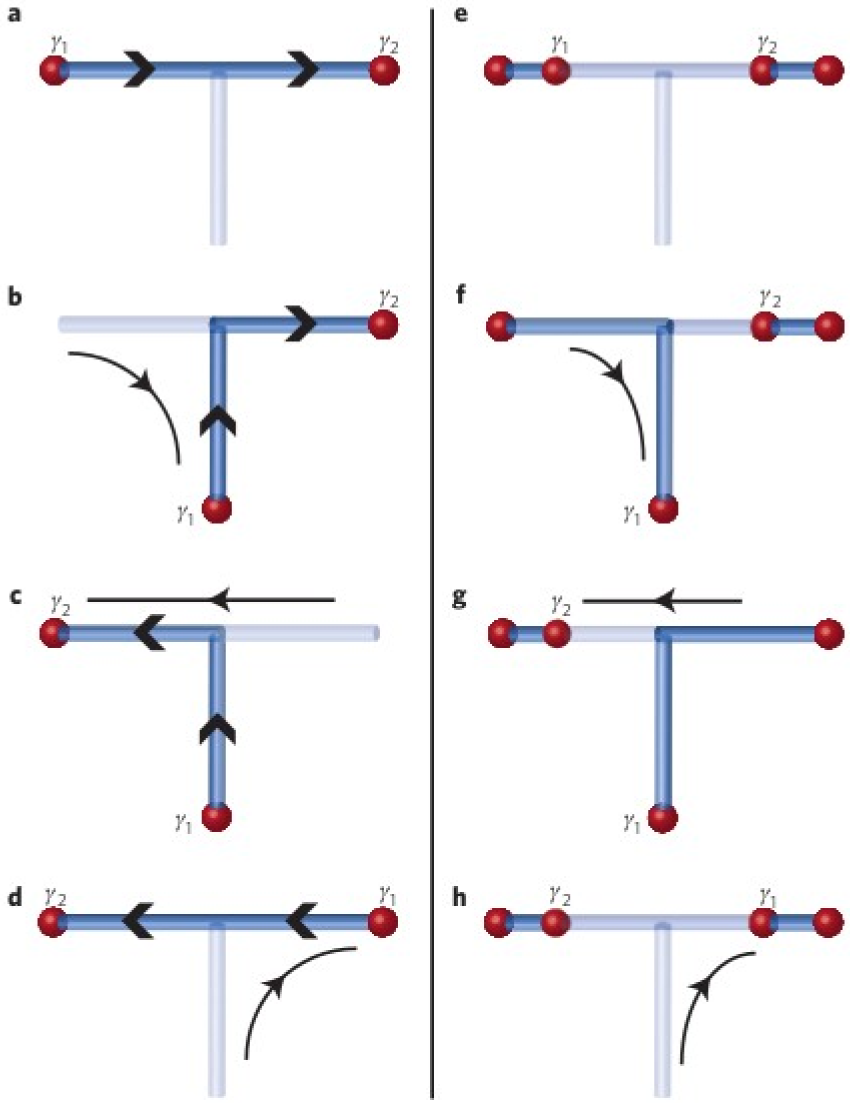
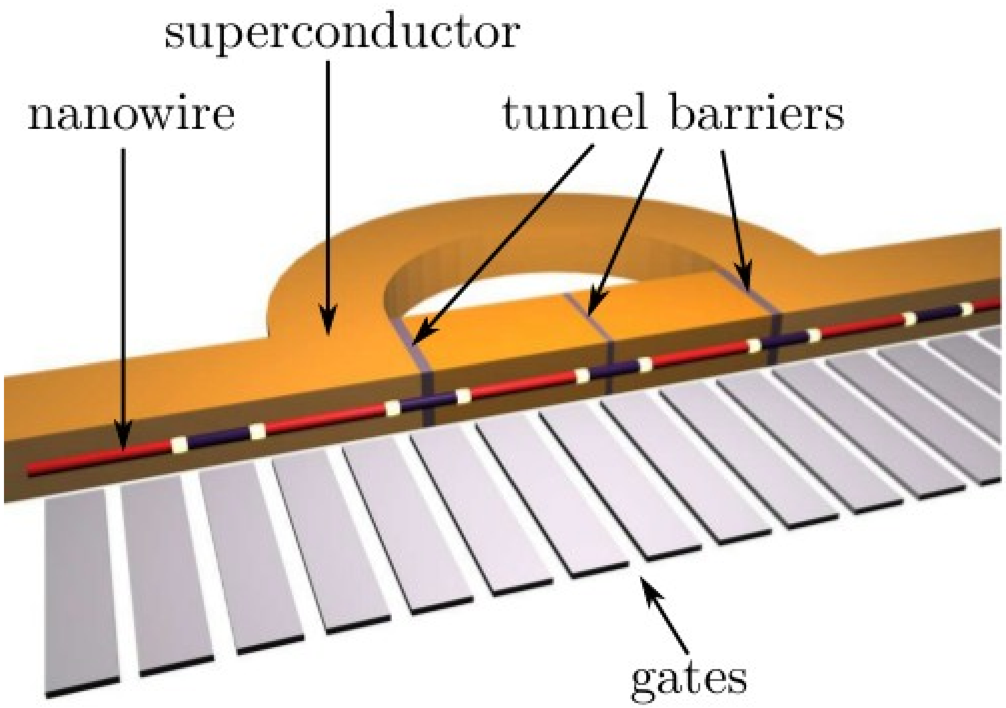


Figure 3 | A T-junction provides the simplest wire network that enables meaningful adiabatic exchange of Majorana fermions. Using the methods

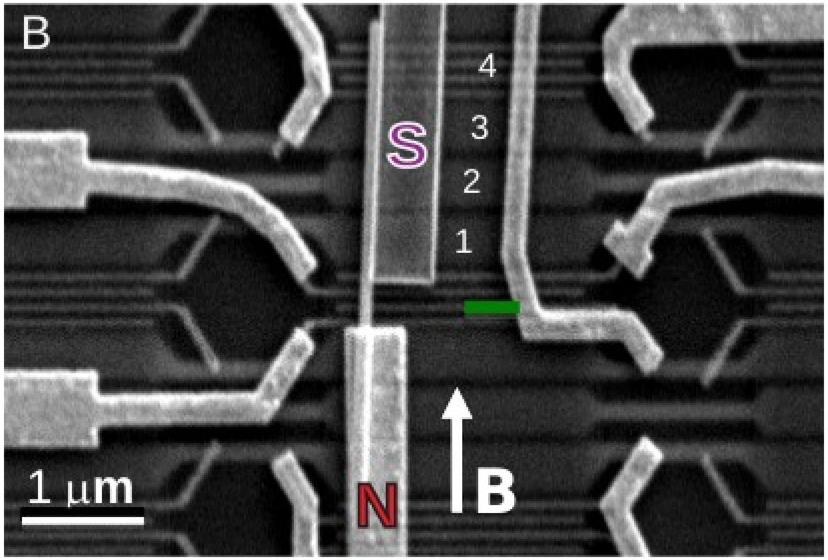
Other operations, readout: Ideas using interferometry, interaction...



[Hassler et al, NJP, 2010]

2012: Experimental race won by Kouwenhoven group, Delft

- Cleanest signs of the presence of protected Majorana states
- No manipulation, no braiding yet
- Most itt posztdokoskodik Geresdi Attila, BME



“Smoking gun”: transmission resonance at 0 energy that appears due to magnetic field

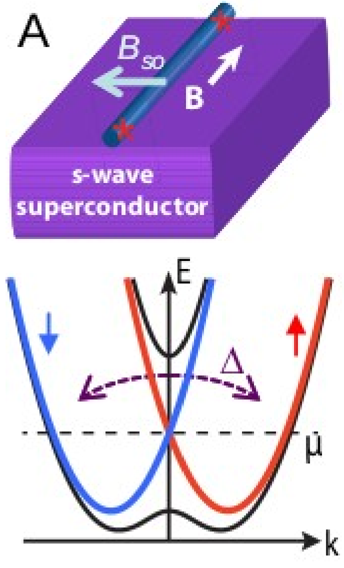
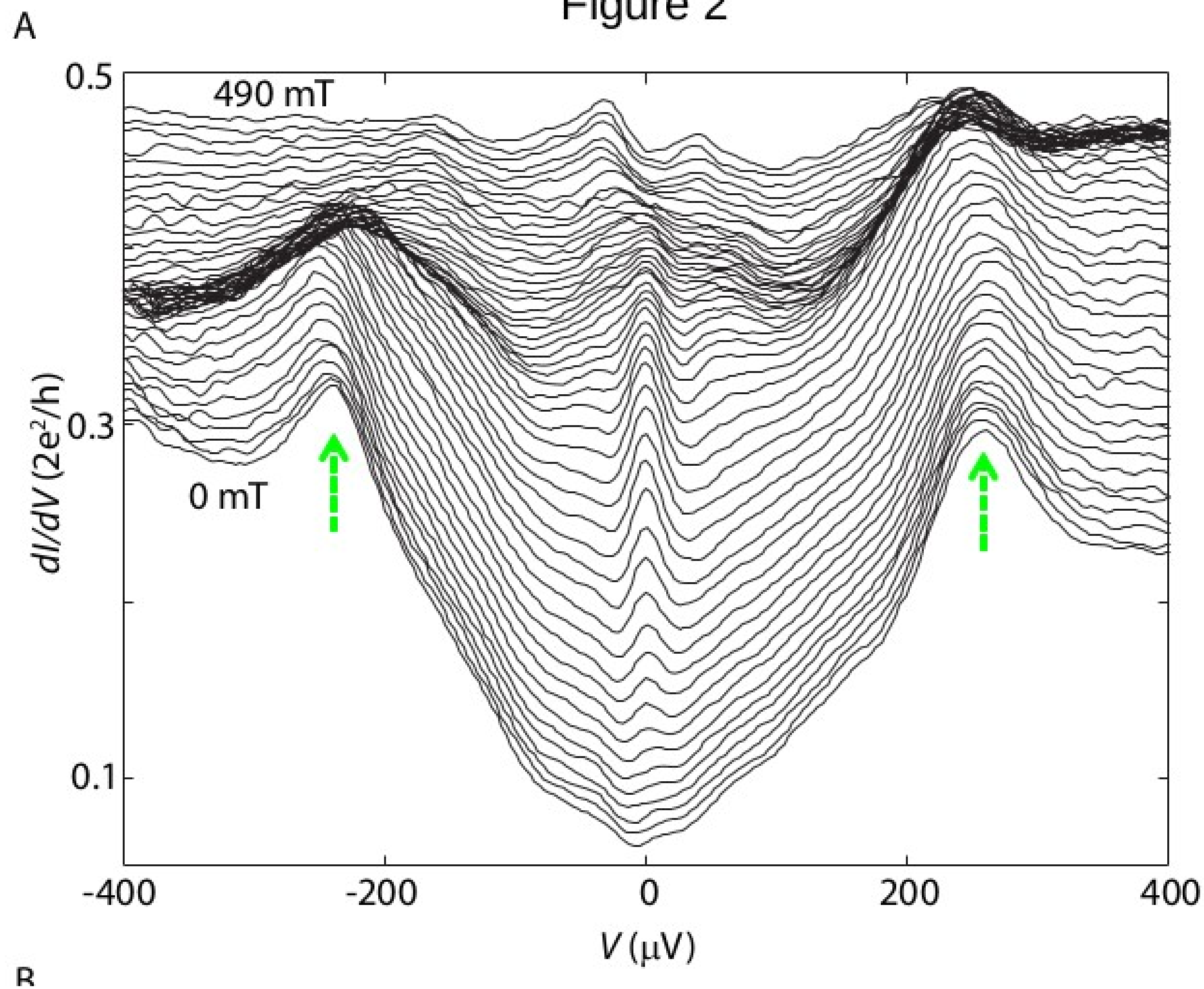
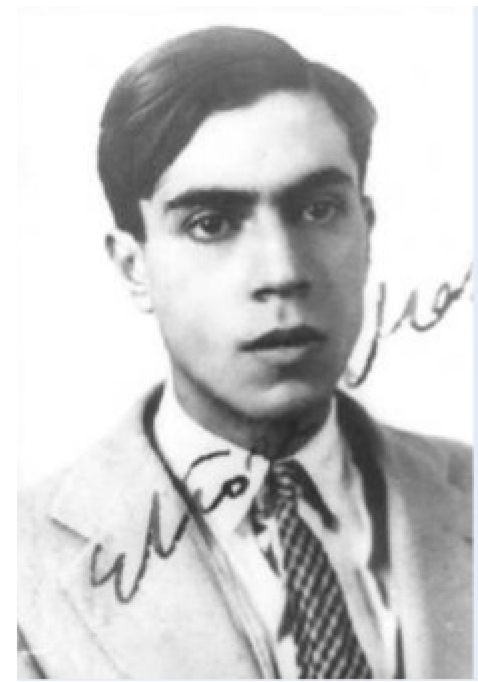


Figure 2



Majorana fermion: 70 year old search

- *1906
- Until 1933: successful physicist, works with Fermi, Heisenberg...
- From 1933: illnesses, no position, no publications
- 1937: Real solutions to Dirac equation, particles can be their own antiparticles
- 1938: boat trip Palermo → Napoli, disappears



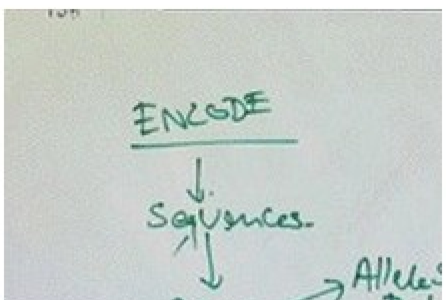
Majorana fermions in quantum wire: not elementary particles, quasiparticles

Almost Breakthrough of the Year 2012 (behind Higgs boson)


Denisovan Genome

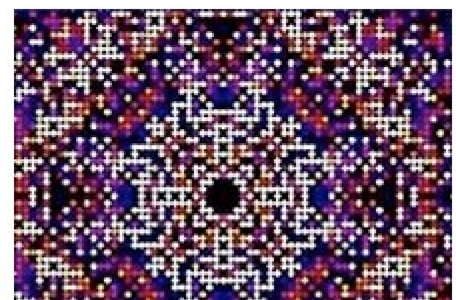

Genome Engineering


Neutrino Mixing Angle

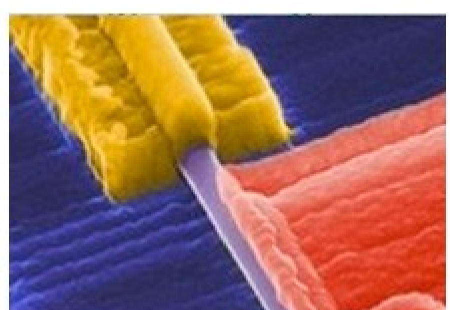


ENCODE


Curiosity Landing


X-ray Laser Advances


Controlling Bionics


Majorana Fermions


Eggs from Stem Cells

Summary

- Environment-induced errors can be prevented by encoding quantum information nonlocally
- Example: 9-bit Shor code
  - Syndrome measurements discretize errors
  - Error correcting operations
- Alternative to error correction is fault-tolerant hardware
- Example: Majorana Wire
  - Qubits protected by particle-hole symmetry and by bulk gap
  - Manipulating nonlocal quantum information: braiding + other ideas also needed
  - Experiment: 1^{st} step = detection of Majorana fermion ready (almost)