## The Black-Hole/Qubit "Correspondence"

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Péter Lévay

February 5, 2014

## BHQC

Recently a striking correspondence has been discovered between two seemingly unrelated fields.
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The basic correspondence is between entropy formulas and classes of black hole solutions in string theory and formulas for entanglement measures and classes of simple multipartite entangled systems with both distinguishable and indistinguishable constituents.

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(2) Black hole solutions in String Theory

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## BHQC

The term Black-Hole/Qubit correspondence has been coined by the string theorist Michael J. Duff.

A recent review of the topic is given in
L. Borsten, M. J. Duff and P. Lévay
"The Black-Hole/Qubit Correspondence: an up-to-date review"
Class. Quantum Grav. 29 (2012) 224008

## Plan of the talk

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(2) Conclusions

## Three qubit entanglement

An arbitrary three-qubit pure state $|\psi\rangle \in \mathbb{C}^{2} \otimes \mathbb{C}^{2} \otimes \mathbb{C}^{2}$ is characterized by 8 complex numbers $\psi_{k j i}$

$$
\left.|\psi\rangle=\sum_{k, j, i=0}^{1} \psi_{k j i}| | k j\right\rangle \quad|k j i\rangle \equiv|k\rangle_{C} \otimes|j\rangle_{B} \otimes|i\rangle_{A}
$$

In a class of quantum information protocols the parties can manipulate their qubits reversibly with some probability of success by performing local manipulations assisted by classical communication between them. Such protocols are called stochastic local operations and classical communication (SLOCC). Mathematically they can be represented as transformations of the form

$$
|\psi\rangle \mapsto(\mathcal{C} \otimes \mathcal{B} \otimes \mathcal{A})|\psi\rangle, \quad \mathcal{C} \otimes \mathcal{B} \otimes \mathcal{A} \in G L(2, \mathbb{C})^{\otimes 3}
$$

Classification of entanglement amounts to classifying the SLOCC orbits.

## The SLOCC classification of three-qubit entanglement

In the physics literature the basic result is due to W. Dür, G. Vidal and J. I. Cirac Phys. Rev. A62 062314 (2000). For the mathematicians this result is known as the classification of trilinear forms for more than 130 years.
C. Le Paige: "Sur les formes trilinéaires", Comptes rendus de I'Académie des Sciences 921103 (1881).
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(3) W-class, eg. $|001\rangle+|010\rangle+|100\rangle$
(0) GHZ-class, eg. $|000\rangle+|111\rangle$

## Cayley's hyperdeterminant as the three-tangle

There are polynomial invariants characterizing these entanglement classes. The most important one is the $S L(2, \mathbb{C})^{\otimes 3}$ and permutation (triality) invariant three-tangle related to Cayley's hyperdeterminant (1845).

\[

\]

The two classes containing genuine tripartite entanglement are the $\mathbf{W}$ and $\mathbf{G H Z}$ classes having $\tau_{A B C}(|W\rangle)=0$ and $\tau_{A B C}(|G H Z\rangle) \neq 0$.

## Attaching special role to qubits

By chosing the first, second or third qubit one can introduce three sets of complex four vectors, e.g. by chosing the first we can define

$$
\xi_{I}^{(A)}=\left(\begin{array}{l}
\psi_{000} \\
\psi_{010} \\
\psi_{100} \\
\psi_{110}
\end{array}\right), \quad \eta_{J}^{(A)}=\left(\begin{array}{l}
\psi_{001} \\
\psi_{011} \\
\psi_{101} \\
\psi_{111}
\end{array}\right) \quad I, J=0,1,2,3
$$

Similarly we can define the four-vectors $\xi^{(B)}, \eta^{(B)}$ and $\xi^{(C)}, \eta^{(C)}$. We also define three bivectors with components called Plücker coordinates

$$
P_{I J}^{(A)}=\xi_{I}^{(A)} \eta_{J}^{(A)}-\xi_{J}^{(A)} \eta_{I}^{(A)}
$$

## The structure of the three-tangle

Then we have

$$
\tau_{A B C}=2\left|P_{I J}^{(A)} P^{(A) I J}\right|=2\left|P_{I J}^{(B)} P^{(B) I J}\right|=2\left|P_{I J}^{(C)} P^{(C) I J}\right|
$$

where indices are raised with respect to the $S L(2, \mathbb{C}) \times S L(2, \mathbb{C})$ invariant metric $g=\varepsilon \otimes \varepsilon$

$$
g^{I J}=\left(\begin{array}{cccc}
0 & 0 & 0 & 1 \\
0 & 0 & -1 & 0 \\
0 & -1 & 0 & 0 \\
1 & 0 & 0 & 0
\end{array}\right)=\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right) \otimes\left(\begin{array}{cc}
0 & 1 \\
-1 & 0
\end{array}\right)
$$

Since the Plücker coordinates are $S L(2, \mathbb{C})$ invariant the expression above shows the $S L(2, \mathbb{C})^{\otimes 3}$ and triality invariance at the same time. Notice that the three-tangle can also be written in the form

$$
\tau_{A B C}=4\left|(\xi \cdot \xi)(\eta \cdot \eta)-(\xi \cdot \eta)^{2}\right|=4 \mid-D(|\psi\rangle) \mid
$$

with $\xi \cdot \eta=g^{I J} \xi_{I} \eta_{J}$.

## The linear entropy

One and two partite reduced density matrices are defined as

$$
\rho_{A}=\operatorname{Tr}_{B C}|\psi\rangle\langle\psi|, \quad \rho_{B C}=\operatorname{Tr}_{A}|\psi\rangle\langle\psi|
$$

Define $\tau_{A(B C)}$ which is two times the linear entropy between the subsystems $A$ and $B C$

$$
\tau_{A(B C)}=4 \operatorname{Det} \rho_{A}=2\left[\operatorname{Tr}\left(\rho_{A}\right)^{2}-1\right]=\sum_{l, J=1}^{4} \bar{P}_{l J}^{(A)} P_{l J}^{(A)}
$$

We can alternatively write

$$
\tau_{A(B C)}=4\left(\langle\xi \mid \xi\rangle\langle\eta \mid \eta\rangle-|\langle\xi \mid \eta\rangle|^{2}\right) \leq 1
$$

$\tau_{A(B C)}=0$ if and only if $\xi^{(A)}$ and $\eta^{(A)}$ are linearly dependent. In this case the corresponding reduced density matrix $\rho_{A}$ has rank one a condition equivalent to $A(B C)$ separability.

## Two-partite correlations inside a three-qubit state

A useful measure for the two-qubit mixed-state entanglement is the Wootters concurrence

$$
\tau_{A B}=\left(\max \left\{\lambda_{1}-\lambda_{2}-\lambda_{3}-\lambda_{4}, 0\right\} \leq 1\right)^{2}
$$

where $\lambda_{i}, i=1,2,3,4$ is the nonincreasing sequence of the square-roots of the eigenvalues for the nonnegative matrix

$$
\rho_{A B} \tilde{\rho}_{A B} \equiv \rho_{A B}(\varepsilon \otimes \varepsilon) \bar{\rho}_{A B}(\varepsilon \otimes \varepsilon)
$$

The invariants discussed above are not independent, they are subject to the important relations

$$
\tau_{A(B C)}=\tau_{A B}+\tau_{A C}+\tau_{A B C}
$$

## The Coffman-Kundu-Wootters relation (2000)

We can write the above relation as

$$
\tau_{A B}+\tau_{A C} \leq \tau_{A(B C)}
$$

A consequence of this is that if two qubits are maximally entangled with each other then neither of them can be at all entangled with the third one. This fact is called the monogamy of entanglement. According to T. J. Osborne and F. Verstraete (2006) this relation also holds for an arbitrary number of qubits a special case is e.g.

$$
\tau_{A B_{1}}+\tau_{A B_{2}}+\ldots \tau_{A B_{n}} \leq 1
$$

Hence qubit $A$ has a limited amount of entanglement to share. Any amount of entanglement that it has with qubit $B_{1}$ reduces the amount available for the rest of the qubits.

## Principal null directions

Consider a pure three-qubit state which is not $A(B C)$ or $(A)(B)(C)$ separable. One can then find null vectors called principal null directions of the form

$$
u_{I} \equiv a \xi_{I}+b \eta_{I}, \quad a, b \in \mathbb{C}
$$

with $a, b$ satisfying

$$
a^{2}(\xi \cdot \xi)+2 a b(\xi \cdot \eta)+b^{2}(\eta \cdot \eta)=0
$$

The PNDs are:

$$
u_{l}=-P_{I J} \xi^{J}-\sqrt{D} \xi_{l}, \quad v_{I}=-P_{I J} \eta^{J}+\sqrt{D} \eta_{I}
$$

For the W-class we have one, for the GHZ-class we have two PNDs. It can be shown that any state $|\psi\rangle$ from the GHZ-class can be brought to the form

$$
|\psi\rangle=|u\rangle \otimes|u\rangle \otimes|u\rangle+|v\rangle \otimes|v\rangle \otimes|v\rangle
$$

where $|u\rangle$ and $|v\rangle$ are determined by the PNDs.

## Real states

The new feature of the classification of SLOCC entanglement types for three "rebits" under the group $G L(2, \mathbb{R})^{\otimes 3}$ is that the usual GHZ-class splits into two classes with representatives

$$
\begin{gathered}
|G H Z\rangle_{-}=\frac{1}{2}(|000\rangle-|011\rangle-|101\rangle-|110\rangle), \quad D<0 \\
|G H Z\rangle_{+}=\frac{1}{2}(|000\rangle+|011\rangle+|101\rangle+|110\rangle), \quad D>0 \\
|G H Z\rangle_{+}=(H \otimes H \otimes H)|G H Z\rangle, \quad H=\frac{1}{\sqrt{2}}\left(\begin{array}{cc}
1 & 1 \\
1 & -1
\end{array}\right) \\
|G H Z\rangle_{-}=(J \otimes J \otimes J)|G H Z\rangle, \quad J=\frac{1}{\sqrt{2}}\left(\begin{array}{cc}
1 & i \\
1 & -i
\end{array}\right) \\
|G H Z\rangle=\frac{1}{\sqrt{2}}(|000\rangle+|111\rangle)
\end{gathered}
$$

## Black Holes

$$
I_{\text {Planck }}^{2}=\frac{\hbar G_{N}}{c^{3}} \simeq 10^{-33} \mathrm{~cm}, \quad m_{\text {Planck }}^{2}=\frac{\hbar c}{G_{N}}
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(0) In this spirit, here the BHs to be considered are massive particle like states of a suitable field theory. Black Holes as supersymmetric solitons.
(6) The BHs we consider are extremal, charged black holes. Extremal means that the Hawking temperature of such objects is zero.

## Black Hole Entropy

The Bekenstein-Hawking entropy formula

$$
S=k \frac{A}{4 I_{\text {Planck }}^{2}}, \quad I_{\text {Planck }}^{2}=\frac{\hbar G_{N}}{c^{3}}
$$

enjoys the following properties:
(1) Maximality

Discreteness means that adding one bit of information will increase the area of the horizon of any black hole by one Planck unit of area. "Information equals area." The horizon can be imagined as a surface, packed densely with "material" bits.

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(9) "No hair" (M,Q,(P),J)

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## Microstates from String Theory

If we tile the horizon with Planck sized cells and assign one degree of freedom to each cell, then $S$ will go like area. This suggests that $S$ can be described by some sort of microstates living on the horizon itself.
Universality of black hole entropy in General Relativity could be an indication of universality in statistical mechanics.

$$
S=k \log \{\text { microstates }\}
$$

The black hole is a strongly coupled quantum system of extended objects (membranes, strings). Note: the specific details of string theory are not needed to derive the area law! One can argue that by making educated guesses on the possible form of some consistent theory of quantum gravity is enough for understanding the microscopic origin of black hole entropy.


String-theory

Buddha: Udana VI. 4

heterotic-e

$$
1995 \rightarrow ?=\text { M-THEORY }
$$

## String compactifications

(1) M-theory
$\mathbf{1} \mapsto \mathbf{2}$ The role of Dualities
$\mathbf{2} \mapsto \mathbf{3}$ The role of Effective Field Theories. From extended objects (branes) to pointlike ones. $3 \mapsto 4$ The role of Compactification. $M_{10}=M \times K$.

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The torus $T^{2}$ arising from a lattice of $\mathbb{C}$


## Complex structure and Kähler structure deformations



## The attractor geometry



## The IIB "duality frame"

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$\mathbf{2} \mapsto \mathbf{3}$ We take the tree level two derivative low energy action of the massless fields of IIB String Theory. This means that we consider the effective action for these fields to leading order in the string coupling constant $\left(g_{s}\right)$ and the inverse string tension $\left(\alpha^{\prime}\right)$.
$\mathbf{3} \mapsto \mathbf{4}$ We deduce the $4 D$ massless spectrum of type IIB String Theory after compactification on a six dimensional space $K$. Now we take $K=T^{6}$. The $10 D$ fields produce $4 D$ descendants by decomposing them according to the harmonic forms basis determined by the geometry of $K$.

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## Effective low energy 4D field content in the IIB picture

Geometry: $M \times K$ now : $K=T^{2} \times T^{2} \times T^{2}$ or $T^{6}$
(1) Coordinates of $M: \xi^{1}, \xi^{2}, \xi^{3}, \xi^{4}$

$$
h^{2,1} \equiv \operatorname{dim} H^{2,1}(K, \mathbb{C})
$$

For $T^{2} \times T^{2} \times T^{2}$ and $T^{6}$ respectively we have

$$
h^{2,1}=3, \quad h^{2,1}=9
$$

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(1) Coordinates of $M: \xi^{1}, \xi^{2}, \xi^{3}, \xi^{4}$
(2) $g_{\mu \nu}(\xi) \leftrightarrow$ describing the $4 D$ spacetime geometry of $M$

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## Effective low energy 4D field content in the IIB picture

Geometry: $M \times K$ now : $K=T^{2} \times T^{2} \times T^{2}$ or $T^{6}$
(1) Coordinates of $M: \xi^{1}, \xi^{2}, \xi^{3}, \xi^{4}$
(2) $g_{\mu \nu}(\xi) \leftrightarrow$ describing the $4 D$ spacetime geometry of $M$
(3) $z^{a}(\xi), a=1, \ldots h^{2,1}(K) \leftrightarrow$ volume preserving fluctuations of K

$$
h^{2,1} \equiv \operatorname{dim} H^{2,1}(K, \mathbb{C})
$$

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(6) $\mathcal{F}_{\mu \nu}^{\prime}(\xi), I=0,1, \ldots h^{2,1}(K) \leftrightarrow$ Maxwell-type fields
(6) $\mathcal{N}_{I J}(\tau(\xi)) \leftrightarrow$ coupling depending on the deformation fields

$$
h^{2,1} \equiv \operatorname{dim} H^{2,1}(K, \mathbb{C})
$$

For $T^{2} \times T^{2} \times T^{2}$ and $T^{6}$ respectively we have

$$
h^{2,1}=3, \quad h^{2,1}=9
$$

For the STU model we have merely three scalar fields and four Maxwell type fields $\mathcal{F}_{\mu \nu}^{\prime}(\xi)(I=0,1,2,3)$.

$$
z_{a}(\xi)=x_{a}(\xi)-i y_{a}(\xi), \quad y_{a}(\xi)>0, \quad a=1,2,3
$$

These scalar fields are sometimes called $S, T, U$ fields, hence the name of the model.

$$
\begin{aligned}
\mathcal{S} & =\frac{1}{8 \pi G_{N}} \int d^{4} \xi \sqrt{|g|}\left\{-\frac{R}{2}+G_{a \bar{b}} \partial_{\mu} z^{a} \partial_{\nu} \bar{z}^{\bar{b}} g^{\mu \nu}\right. \\
& \left.+\mathcal{I}_{I J} \mathcal{F}_{\mu \nu}^{\prime} \mathcal{F}^{J \mu \nu}+\mathcal{R}_{I J} \mathcal{F}_{\mu \nu}^{\prime}{ }^{*} \mathcal{F}^{J \mu \nu}\right\}+\ldots
\end{aligned}
$$

$$
\begin{gathered}
G_{a \bar{b}}=\frac{\delta_{a \bar{b}}}{\left(2 y_{a}\right)^{2}} \\
\mathcal{R}_{I J}=\left(\begin{array}{cccc}
2 x_{1} x_{2} x_{3} & -x_{2} x_{3} & -x_{1} x_{3} & -x_{1} x_{2} \\
-x_{2} x_{3} & 0 & x_{3} & x_{2} \\
-x_{1} x_{3} & x_{3} & 0 & x_{1} \\
-x_{1} x_{2} & x_{2} & x_{1} & 0
\end{array}\right), \\
\mathcal{I}_{I J}=-y_{1} y_{2} y_{3}\left(\begin{array}{ccccc}
1+\left(\frac{x_{1}}{y_{1}}\right)^{2}+\left(\frac{x_{2}}{y_{2}}\right)^{2}+\left(\frac{x_{3}}{y_{3}}\right)^{2} & -\frac{x_{1}}{y_{1}^{2}} & -\frac{x_{2}}{y_{2}^{2}} & -\frac{x_{3}}{y_{3}^{2}} \\
-\frac{x_{1}}{y_{1}^{2}} & \frac{1}{y_{1}^{2}} & 0 & 0 \\
-\frac{x_{2}}{y_{2}^{2}} & 0 & \frac{1}{y_{2}^{2}} & 0 \\
-\frac{x_{3}}{y_{3}^{2}} & 0 & 0 & \frac{1}{y_{3}^{2}}
\end{array}\right)
\end{gathered}
$$

Our aim is to solve the Euler-Lagrange equations arising from this action under special conditions. $\mapsto$ Black Hole Solutions

## Extremal Black Hole Solutions

The solutions we are searching for are
(1) Static

The solutions we find are of extremal Reissner-Nordström type. Supersymmetric solitons.

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(9) Extremal

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## Extremal Black Hole Solutions

The solutions we are searching for are
(1) Static
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(3) Asymptotically Minkowski
(9) Extremal
© Supersymmetric (BPS)
The solutions we find are of extremal Reissner-Nordström type. Supersymmetric solitons.

If we take the ansatz for the space-time metric

$$
d s^{2}=g_{\mu \nu}(\xi) d \xi^{\mu} d \xi^{\nu}=-e^{2 U(r)} d t^{2}+e^{-2 U(r)}\left(d r^{2}+r^{2} d \Omega^{2}\right)
$$

and introduce a spherically symmetric ansatz also for the gauge-fields $\mathcal{F}^{\prime}$ with electric and magnetic charges $q_{I}$ and $p^{\prime}$ and employing the new variable $\varrho \equiv \frac{1}{r}$ we get the action ( $T \equiv \int d t$ is the elapsed time and dot denotes $\frac{d}{d \varrho}$ )

$$
\begin{gathered}
S_{4 D} / T=\frac{1}{2 G_{N}} \int_{0}^{\infty} d \varrho\left(\dot{U}^{2}+G_{a b} \dot{z}^{a^{\circ} \dot{\bar{z}}^{\bar{b}}}+G_{N} e^{2 U} V_{B H}\right) \\
V_{B H}=\frac{1}{2}\left(\begin{array}{ll}
p^{\prime} & q_{I}
\end{array}\right)\left(\begin{array}{cc}
\left(\mathcal{R}+\mathcal{R} \mathcal{I}^{-1} \mathcal{R}\right)_{I J} & -\left(\mathcal{R} \mathcal{I}^{-1}\right)_{I} \\
-\left(\mathcal{I}^{-1} \mathcal{R}\right)_{J}^{\prime} & \left(\mathcal{I}^{-1}\right)^{\prime J}
\end{array}\right)\binom{p^{J}}{q_{J}} .
\end{gathered}
$$

We also have the constraint.

$$
\dot{U}^{2}+G_{a b} \dot{z}^{a^{\prime}} \dot{\bar{z}}^{\bar{b}}-G_{N} e^{2 U} V_{B H}=0
$$

## Equations of motion for the scalar fields

The equations derived from this effective action describe the RADIAL dynamics of the space-time warp factor and the fluctuating extra dimensions in the near horizon $\varrho \rightarrow \infty$ limit. Extremization of the effective Lagrangian with respect to the warp factor and the scalar fields yields the Euler-Lagrange equations

$$
\ddot{U}=G_{N} e^{2 U} V_{B H}, \quad \ddot{z}^{a}+\Gamma_{b c}^{a} \dot{z}^{b} \dot{z}^{c}=G_{N} e^{2 U} \partial^{a} V_{B H} .
$$

In these equations the dots denote derivatives with respect to $\varrho$. These radial evolution equations taken together with the constraint determine the structure of static spherically symmetric extremal black hole solutions in the STU model.

## Supersymmetric (BPS) solutions

Note that due to supersymmetry we have also fermion fields in the effective Lagrangian which is supersymmetric under transformations that can be written schematically as

$$
\delta B=f(F), \quad \delta F=g(B)
$$

For classical solutions we have to set $F=0$. However, this not makes $\delta F=0$. If we also impose this condition then this yield a further constraint on the $B$ bosonic fields. Such solutions of the Euler Lagrange equations are called supersymmetric.

Define the central charge $\mathcal{Z}$
$\mathcal{Z}=\frac{1}{\sqrt{8 y^{1} y^{2} y^{3}}}\left(q_{0}+z^{a} q_{a}-z^{1} z^{2} p^{3}-z^{2} z^{3} p^{1}-z^{3} z^{1} p^{2}+z^{1} z^{2} z^{3} p^{0}\right)$

## Supersymmetric (BPS) solutions. Attractors.

S. Ferrara and R. Kallosh, Phys. rev. D54, 1514 (1996)

By virtue of SUSY the equations of motion will be of first order

$$
\begin{gathered}
\dot{U}=-\sqrt{G_{N}} e^{U}|\mathcal{Z}| \\
\dot{z}^{a}=-2 \sqrt{G_{N}} e^{U} G^{a \bar{b}} \partial_{\bar{b}}|\mathcal{Z}|
\end{gathered}
$$

Then solutions of these equations (our black hole solutions) are supersymmetric solitons.
One shows that $|\mathcal{Z}(\varrho)|$ is a monotonically decreasing function converging to a minimum. The fixed point is determined from

$$
|\dot{\mathcal{Z}}| \rightarrow 0, \quad \text { as } \quad \varrho \rightarrow \infty
$$

Assuming that $\lim _{\varrho \rightarrow \infty} \mathcal{Z}(\varrho) \equiv \mathcal{Z}_{*} \neq 0$ we get

$$
\varrho^{-1} e^{-U(\varrho)} \rightarrow \sqrt{G_{N}}\left|\mathcal{Z}_{*}\right|
$$

## The near horizon geometry

Since as $r \rightarrow 0$ we have $e^{-U} \rightarrow \sqrt{G_{N}}\left|\mathcal{Z}_{*}\right| / r$ then the near horizon geometry from

$$
d s^{2}=-e^{2 U} d t^{2}+e^{-2 U}\left(d r^{2}+r^{2} d \Omega^{2}\right)
$$

is

$$
d s^{2} \simeq-\frac{r^{2}}{G_{N}\left|\mathcal{Z}_{*}\right|^{2}} d t^{2}+\frac{G_{N}\left|\mathcal{Z}_{*}\right|^{2}}{r^{2}} d r^{2}+G_{N}\left|\mathcal{Z}_{*}\right|^{2} d \Omega^{2}
$$

Let $R^{2} \equiv G_{N}\left|\mathcal{Z}_{*}\right|^{2}$ and $u \equiv R^{2} / r$ then we get

$$
d s^{2}=R^{2}\left(\frac{-d t^{2}+d u^{2}}{u^{2}}\right)+R^{2} d \Omega^{2}
$$

Hence the near horizon geometry is $A d S_{2} \times S^{2}$ with the radius of curvature of both spaces are the same up to a sign.

## The macroscopic Black Hole Entropy

$$
\begin{gathered}
R^{2}=G_{N}\left|\mathcal{Z}_{*}\right|^{2} \\
S=\frac{A}{4 G_{N}}=\frac{4 \pi R^{2}}{4 G_{N}}=\pi\left|\mathcal{Z}_{*}\right|^{2}
\end{gathered}
$$

Note: due to supersymmetry we have the BPS bound $G_{N} M_{B P S}=\left|\mathcal{Z}_{*}\right|$, and extremality is related to the saturation of the BPS condition

$$
\sqrt{G_{N}} M_{B P S}=\left|\mathcal{Z}_{*}\right|
$$

Note: $\mathcal{Z}_{*}$ is depending on the charges $p^{\prime}, q_{l}$ and the horizon values of the scalar fields i.e. $z^{a}(0)$. The charges are quantized, however the $z^{a}(0)$ can be changed continuously. It is an undesirable thing if we would like to interpret $S$ as an object "counting states". Hence it is desirable to find a means of relating somehow $z^{a}(0)$ to the charges.

## The Attractor Mechanism. Attractor Equations.

It can be shown that in the supersymmetric case the solutions of the equations of motion i.e. the flow

$$
z^{a}(r), \quad r \in[0, \infty], \quad a=1,2,3
$$

describes an attractor in moduli space. This means that independent of the asymptotic values of the scalar fields, i.e. the values $z^{a}(\infty)$, the solutions flow to the attractor value

$$
z_{*}^{a}\left(q_{I}, p^{\prime}\right) \equiv z^{a}(0)
$$

obtained from the extremization of the BPS mass with respect to the scalar fields

$$
\frac{\partial}{\partial z^{a}}|\mathcal{Z}(r)|=0
$$

As a result of this the black hole entropy will be a function of the charges.

## The attractor geometry



## Defining a charge dependent three-qubit state

M. J. Duff: Phys. Rev. D76 025017 (2007)

$$
\begin{gathered}
|\Gamma\rangle=\sum_{k, j, i=0,1} \Gamma_{k j i}|k j i\rangle, \quad|k j i\rangle \equiv|k\rangle_{U} \otimes|j\rangle_{T} \otimes|i\rangle_{S} \\
\left(\begin{array}{ccc}
p^{0}, & p^{1}, & p^{2}, \\
-p^{3} \\
-q_{0}, & q_{1}, & q_{2}, \\
q_{3}
\end{array}\right)=\left(\begin{array}{lll}
\Gamma_{000}, & \Gamma_{001}, & \Gamma_{010}, \\
\Gamma_{111}, & \Gamma_{110}, & \Gamma_{101}, \\
\Gamma_{011}
\end{array}\right)
\end{gathered}
$$

The main observation then was that for supersymmetric black holes in units $\hbar=c=k=1$ we have

$$
S=\pi \sqrt{-D(\Gamma)}
$$

where $D$ is Cayleys hyperdeterminant known from three-qubit entanglement.

## An example for a charge combination which is not SUSY (NBPS)

R. Kallosh and A. Linde, Phys. Rev. D73 104033 (2006)

$$
\begin{equation*}
|\Gamma\rangle=|000\rangle+|111\rangle, \quad p^{0}=1, q_{0}=-1 \tag{NBPS}
\end{equation*}
$$

$$
\begin{equation*}
|\Gamma\rangle=|000\rangle-|011\rangle-|101\rangle-|110\rangle),-p^{0}=q_{1}=q_{2}=q_{3}=-1 \tag{BPS}
\end{equation*}
$$

$$
\begin{aligned}
D(|\psi\rangle) & \equiv \psi_{000}^{2} \psi_{111}^{2}+\psi_{001}^{2} \psi_{110}^{2}+\psi_{010}^{2} \psi_{101}^{2}+\psi_{011}^{2} \psi_{100}^{2} \\
& -2\left(\psi_{000} \psi_{001} \psi_{110} \psi_{111}+\psi_{000} \psi_{010} \psi_{101} \psi_{111}\right. \\
& +\psi_{000} \psi_{011} \psi_{100} \psi_{111}+\psi_{001} \psi_{010} \psi_{101} \psi_{110} \\
& \left.+\psi_{001} \psi_{011} \psi_{110} \psi_{100}+\psi_{010} \psi_{011} \psi_{101} \psi_{100}\right) \\
& 4\left(\psi_{000} \psi_{011} \psi_{101} \psi_{110}+\psi_{001} \psi_{010} \psi_{100} \psi_{111}\right)
\end{aligned}
$$

## Real states. STU: on shell $S L(2, \mathbb{R})^{\otimes 3}$ symmetry!

The new feature of the classification of SLOCC entanglement types for three "rebits" under the group $G L(2, \mathbb{R})^{\otimes 3}$ is that the usual GHZ-class splits into two classes with representatives

$$
\begin{aligned}
&|G H Z\rangle_{-}= \frac{1}{2}(|000\rangle-|011\rangle-|101\rangle-|110\rangle), \quad D<0 \\
&|G H Z\rangle_{+}= \frac{1}{2}(|000\rangle+|011\rangle+|101\rangle+|110\rangle), \quad D>0 \\
&|G H Z\rangle_{+}=(H \otimes H \otimes H)|G H Z\rangle, \quad H=\frac{1}{\sqrt{2}}\left(\begin{array}{cc}
1 & 1 \\
1 & -1
\end{array}\right) \\
&|G H Z\rangle_{-}=(J \otimes J \otimes J)|G H Z\rangle, \quad J=\frac{1}{\sqrt{2}}\left(\begin{array}{cc}
1 & i \\
1 & -i
\end{array}\right) \\
&|G H Z\rangle=\frac{1}{\sqrt{2}}(|000\rangle+|111\rangle)
\end{aligned}
$$

## Defining a charge and moduli dependent three-qubit

 state"P. L.: Phys. Rev. D74 024030 (2006)

Let

$$
\begin{aligned}
\mathcal{S}_{a} & \equiv \frac{1}{\sqrt{2 y^{a}}}\left(\begin{array}{cc}
\bar{z}^{a} & -1 \\
-z^{a} & 1
\end{array}\right) \\
=\mathcal{U} S_{a} & \equiv \frac{1}{\sqrt{2}}\left(\begin{array}{cc}
i & -1 \\
i & 1
\end{array}\right) \frac{1}{\sqrt{y^{a}}}\left(\begin{array}{cc}
y^{a} & 0 \\
-x^{a} & 1
\end{array}\right),
\end{aligned}
$$

then define a new three-qubit "state" as

$$
\begin{aligned}
|\psi(r)\rangle & =\left(\mathcal{S}_{3}(r) \otimes \mathcal{S}_{2}(r) \otimes \mathcal{S}_{1}(r)\right)|\Gamma\rangle \\
& =(\mathcal{U} \otimes \mathcal{U} \otimes \mathcal{U})\left(S_{3}(r) \otimes S_{2}(r) \otimes S_{1}(r)\right)|\Gamma\rangle
\end{aligned}
$$

Now

$$
V_{B H}(r)=\frac{1}{2}\|\psi(r)\|^{2}
$$

## BPS STU Black Holes and three-qubits

Note: for $r \rightarrow \infty$ we have Minkowski space-time (asymptotically flat limit)
for $r \rightarrow 0$ we have $\operatorname{AdS}_{2} \times S^{2}$ space-time (near horizon limit)
The GHZ components of $|\psi(r)\rangle$ have special meaning as the central charge

$$
\begin{gathered}
\psi_{000}(\infty)=-\overline{\psi_{111}}(\infty)=\overline{\mathcal{Z}} \\
\left|\psi_{000}(0)\right|^{2}=\left|\psi_{111}(0)\right|^{2}=G_{N} M_{B P S}^{2}
\end{gathered}
$$

Since our black hole solution is a soliton interpolating between two max. supersymmetric vacuum solutions and we also have the saturation of the BPS bound $\sqrt{G_{N}} M_{B P S}=|\mathcal{Z}|$, we can regard $M_{B P S}$ as the mass of the black hole.

Let us define

$$
\rho=\frac{1}{2}(|000\rangle\langle 000|+|111\rangle\langle 111|)
$$

then

$$
G_{N} M_{B P S}^{2}=\langle\rho\rangle_{\psi_{*}} \equiv\left\langle\psi_{*}\right| \rho\left|\psi_{*}\right\rangle
$$

where for the calculation of $\left|\psi_{*}\right\rangle \equiv|\psi(0)\rangle$ the $z_{*}^{a} \equiv z^{a}(0)$ attractor values are needed

$$
z_{*}^{a}=\frac{(\xi \cdot \eta)^{a}-i \sqrt{-D(\Gamma)}}{(\xi \cdot \xi)^{a}}
$$

Plugging in these values we get for the entropy

$$
S=\pi\langle\rho\rangle_{\psi_{*}}=\pi \sqrt{-D(\Gamma)}
$$

"Distillation" of GHZ-like "states"? $\leftrightarrow$ Attractors?

$$
|\psi(0)\rangle=(-D(\Gamma))^{1 / 4}\left(e^{i \alpha}|000\rangle-e^{-i \alpha}|111\rangle\right)
$$

$$
\tan \alpha=\sqrt{-D(\Gamma)} \frac{p^{0}}{2 p^{1} p^{2} p^{3}+p^{0}\left(p^{0} q_{0}+p^{1} q_{1}+p^{2} q_{2}+p^{3} q_{3}\right)}=\frac{p^{0}}{\hat{p^{0}}}
$$

This is a GHZ-like state.
Note, that this is valid merely for BPS black holes for which $D<0$. For non BPS ones the states on the horizon are special cases of graph states. They are again complex ones but satisfying reality conditions corresponding to the real class with $D>0$. Then

$$
S=\pi \sqrt{|D(\Gamma)|}
$$

## Attaching special role to qubits again

By chosing the first, second or third qubit one can introduce three sets of real four vectors, e.g. by chosing the first we can define

$$
\begin{gathered}
\xi_{l}^{(1)}=\left(\begin{array}{l}
\Gamma_{000} \\
\Gamma_{010} \\
\Gamma_{100} \\
\Gamma_{110}
\end{array}\right), \quad \eta_{J}^{(1)}=\left(\begin{array}{c}
\Gamma_{001} \\
\Gamma_{011} \\
\Gamma_{101} \\
\Gamma_{111}
\end{array}\right) \quad I, J=0,1,2,3 \\
\left(\begin{array}{cccc}
p^{0}, & p^{1}, & p^{2}, & p^{3} \\
-q_{0}, & q_{1}, & q_{2}, & q_{3}
\end{array}\right)=\left(\begin{array}{lll}
\Gamma_{000}, & \Gamma_{001}, & \Gamma_{010}, \\
\Gamma_{111}, & \Gamma_{110}, & \Gamma_{101}, \\
\Gamma_{011}
\end{array}\right) \\
(\xi \cdot \eta)^{1} \equiv \xi_{l}^{(1)} g^{I J} \eta_{J}^{(1)}
\end{gathered}
$$

## Some notation

$$
\begin{gathered}
\mathcal{M}^{a}(r)=\frac{1}{y^{a}(r)}\left(\begin{array}{cc}
1 & x^{a}(r) \\
x^{a}(r) & \left(x^{a}(r)\right)^{2}+\left(y^{a}(r)\right)^{2}
\end{array}\right) \in S L(2, \mathbb{R}) \\
\Gamma^{a}=\frac{1}{\sqrt{-D(\Gamma)}}\left(\begin{array}{ll}
(\xi \cdot \xi)^{a} & (\xi \cdot \eta)^{a} \\
(\xi \cdot \eta)^{a} & (\eta \cdot \eta)^{a}
\end{array}\right) \in S L(2, \mathbb{R})
\end{gathered}
$$

Define

$$
g\left(\mathcal{M}^{a}(r), \Gamma^{a}\right) \equiv-\frac{1}{2} \operatorname{Tr}\left(\mathcal{M}^{a}(r) \sigma_{2}\left\ulcorner\sigma_{2}\right)\right.
$$

then for

$$
M=\left(\begin{array}{cc}
T-X & Y \\
Y & T+X
\end{array}\right)
$$

we have

$$
g(M, M)=X^{2}+Y^{2}-T^{2}
$$

hence the space of such matrices equipped with $g$ becomes isomorphic to $2 \oplus 1$ dimensional Minkowski space.

## Attractors from vanishing concurrence

P. L. and Szilárd Szalay: Phys. Rev. D83, 045005 (2011)

In three-qubit entanglement Cayley's hyperdeterminant is related to the physical notion monogamy and not entropy. We have

$$
\tau_{123}(\Gamma)=4|D(\Gamma)|, \quad S=\pi \sqrt{-D(\Gamma)}
$$

How to tackle this?!

$$
\tau_{a b}(\psi(r))=\tau_{123}(\Gamma)\left(g\left(\mathcal{M}^{c}(r), \Gamma^{c}\right)+1\right)\left(g\left(\mathcal{M}^{c}(r), \Gamma^{c}\right)-1\right)
$$

From this it can be shown that the Wootters concurrence is vanishing if and only if the BPS attractor equations hold. Since according to CKW we have

$$
\tau_{123}=\tau_{1(23)}-\tau_{12}-\tau_{13}, \quad \text { and } \quad \text { cyclic } \quad \text { permutations }
$$

Hence at the horizon $\tau_{123}=\tau_{1(23)}=\tau_{2(13)}=\tau_{3(12)}$. Now it is OK. since $\tau_{a(b c)}$ is the linear entropy (Tsallis entropy).

## BPS attractors



## Non BPS attractors



Main idea: Wrapped membranes around homology cycles of extra dimensions should give rise to qubits.
L. Borsten et. al. Phys. Rev. Lett. 100251602 (2008)
"To wrap or not to wrap that is the qubit" (M. J. Duff).
One can make this idea precise by obtaining simple entangled systems from the cohomology of the extra dimensions.
P. L.: Phys. Rev. D82, 125020 (2011)

Wrapped brane configurations with different winding numbers are known to give rise to charges of both electric and magnetic type in our the $4 D$ low energy world. Such winding configurations account for the microstates of certain elementary black hole solutions.

Homology base


## STU Black Holes as four qubit systems

Can we put in the warp factor $e^{2 U(r)}$ to an extra $S L(2, \mathbb{R})$ ? Yes: provided we are enlarging the allowed set of solutions to stationary, non extremal black holes!
E. Bergshoeff et.al.: Nucl. Phys. B812 343 (2009)
P. L.: Phys. Rev. D82 026003 (2010)

STU black hole solutions in this picture correspond to geodesics on the moduli space $S O(4,4) / S L(2, \mathbb{R})^{\otimes 4}$. Such solutions are classified in terms of conserved Noether charges. It turns out that the classification of Noether charges can be mapped to the classification of complex SLOCC classes for four qubits.

Borsten et.al.: Phys. Rev. Lett. 105100507 (2010)
For the media hype of this paper see the blog etc. links in arXiv:1005.4915
"String Theory Finally Does Something Useful"

## Conclusions. Questions.

(1) Where are the "States"?

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## Conclusions. Questions.

(1) Where are the "States"?
(2) Is it "Quantum"?
(3) Where are the subsystems being "Entangled"?
(9) What is it good for?

## Conclusions. "Answers"

(1)

$$
S=\pi\langle\rho\rangle_{\psi_{*}}
$$

However: Is it a macroscopic manifestation of some genuine degenerate state describing microstates? (ADS/CFT, BH entropy as entanglement entropy etc.)

## Conclusions. "Answers"

(1)

$$
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However: Is it a macroscopic manifestation of some genuine degenerate state describing microstates? (ADS/CFT, BH entropy as entanglement entropy etc.)
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(3) In the STU model three tori are "entangled" . Interestingly the three copies of SLOCC groups are special cases of the so called U-duality groups relating different string theories.
(1) For example for constructing new pure state multipartite measures of entanglement.

