Electron transport in graphene

"Graphene = a material for future electronics"

- Conductivity & mobility in graphene
    - Drude picture
    - Effective mass
    - Boltzmann equation
- Scattering mechanisms, limitation of mobility
- Present status
- Applications

"Dirac physics" in transport

- Klein tunneling
- Reflectionless transmission in p-n junction
- Evidence of Klein backscattering in interference pattern of n-p-n junction

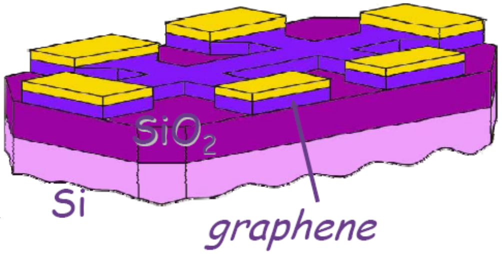
Graphene based Hybrid Quantum Devices

Sources:
http://www.tntconf.org/2010/Presentaciones/TNT2010_Geim.pdf

Conductivity, mobility

$$E(\vec{k}) = \hbar v_F |\vec{k}|$$

Mobility, conductance:



$$v_d \equiv \mu E$$

$$j \equiv env_d = en\mu E = \sigma E \quad \sigma = en\mu$$

Conductivity (Drude model):

$$\sigma = \frac{e^2 n\tau}{m} \qquad \mu = \frac{e\tau}{m}$$

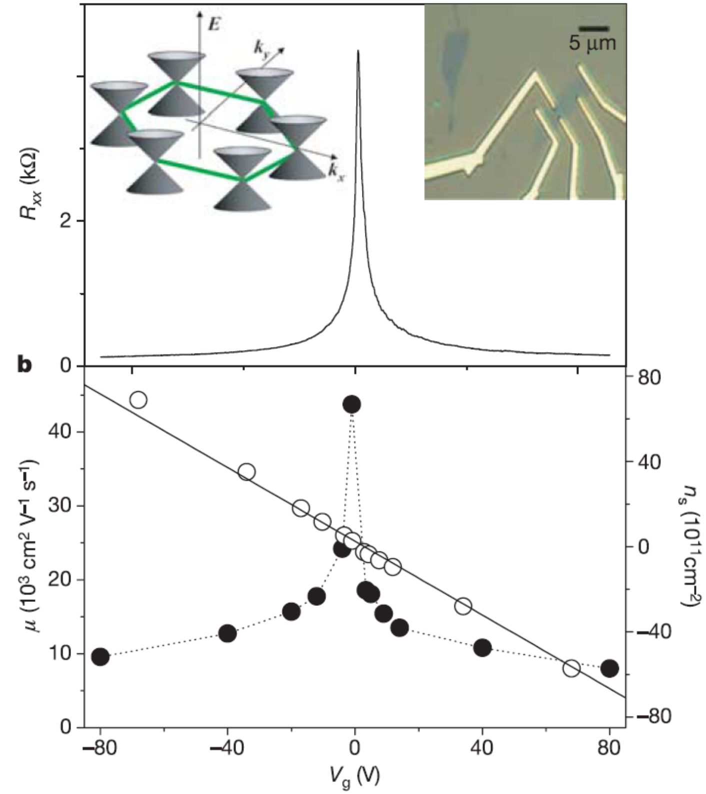
How to separate mobility (μ) and e density (n)?

Measure: ρ + Hall resistance $R_H = \frac{E_y}{j_x B} = -\frac{1}{ne}$

Effect of gate voltage, V_g?

$$N = C_g V_g / e \quad \rightarrow \quad n \sim V_g \quad \rightarrow \quad k_F \sim \sqrt{V_g}$$

R* vs. *V_g *Transport characteristics*



Measurement:
- At V_g zero, R_H (and n) changes sign → boarder between e and h bands
- mobility largest at Dirac point ($V_g = 0$).

Nature 438, 201 (2005)

Conductivity, mobility

$$E(\vec{k}) = \hbar v_F |\vec{k}|$$

How to calculate conductivity?

$$\sigma = e^2 \tau \frac{n}{m}$$

What is m, effective mass?

$$\frac{1}{m} = \frac{1}{m_{xx}} = \frac{1}{\hbar^2}\frac{\partial^2 E}{\partial k_x^2}$$

For quadratic dispersion: $E = \frac{\hbar^2 k^2}{2m_{eff}}, m = m_{eff}$

For Dirac electrons, where $E(\vec{k}) = \hbar v_F |\vec{k}|$?

Naively 1/m= 0, but NOT.

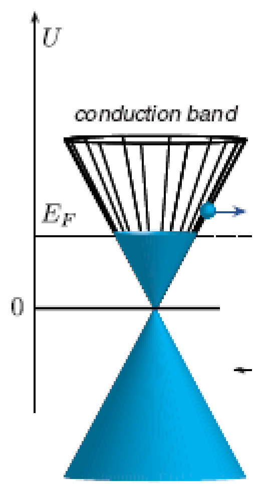
To calculate 1/m:

$$\frac{\partial^2 |k|}{\partial k_x^2} = \cdots = \frac{k_y^2}{|k|^3} \rightarrow \quad \frac{1}{m_{xx}} = \frac{1}{\hbar} v_F \frac{k_y^2}{k^3}$$

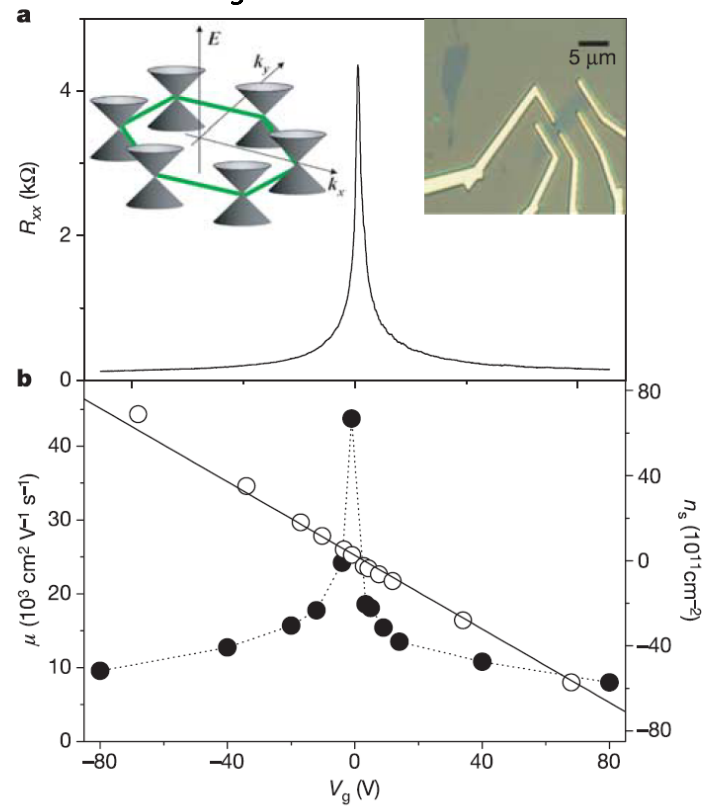
$$\frac{\partial |k|}{\partial k_x} = \frac{1}{2}\frac{2k_x}{|k|}$$

→ Effective mass depends on k

One has to average 1/m for all filled states:



R vs. V_g *Transport characteristics*



Nature 438, 201 (2005)

Conductivity, mobility

$$E(\vec{k}) = \hbar v_F|\vec{k}|$$

Accurate calculation of σ?

From Boltzmann equation (see Solyom 24.3.39.):

$$\sigma = e^2\tau\frac{n}{m} = e^2\tau\cdot 2\cdot 2\cdot\int_{filled\ k\ states}\frac{d^2k}{(2\pi)^2}\frac{1}{m_{xx}}$$

HOMEWORK Calculate σ

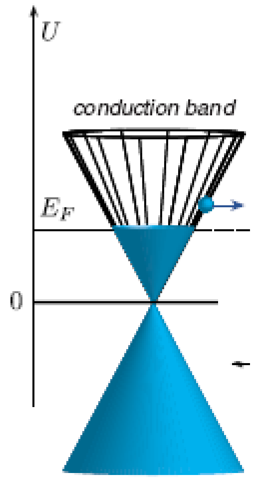
Result: $$\sigma = e^2\tau\frac{v_F}{\hbar\pi}k_F$$

with relaxation length $l \equiv v_F\tau$

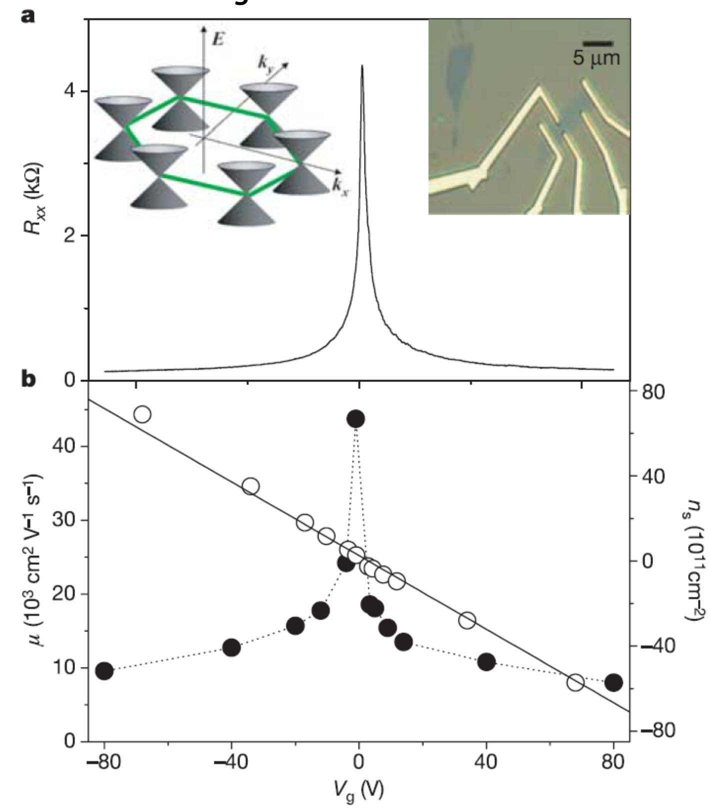
$$\sigma = \frac{2e^2}{h}lk_F$$

Mean free path: $$l = \frac{\hbar}{e}\mu\sqrt{n\pi}$$

E.g. for mobility =600.000, l is only ≈ 3µm



***R* vs. V_g** *Transport characteristics*



Nature 438, 201 (2005)

Scattering mechanisms in graphene

What limits the mobility at room T?

Source of $1/\tau$?

Scattering mechanisms resulting resistivity:

- potential scattering: impurities, defects, vacancies
- Electron – phonon scattering
- Etc.

Usual terms: (see Solyom II.)

- Residual resistivity (ρ_0): T independent
- Longitudinal acoustic phonons (ρ_A): linear in T

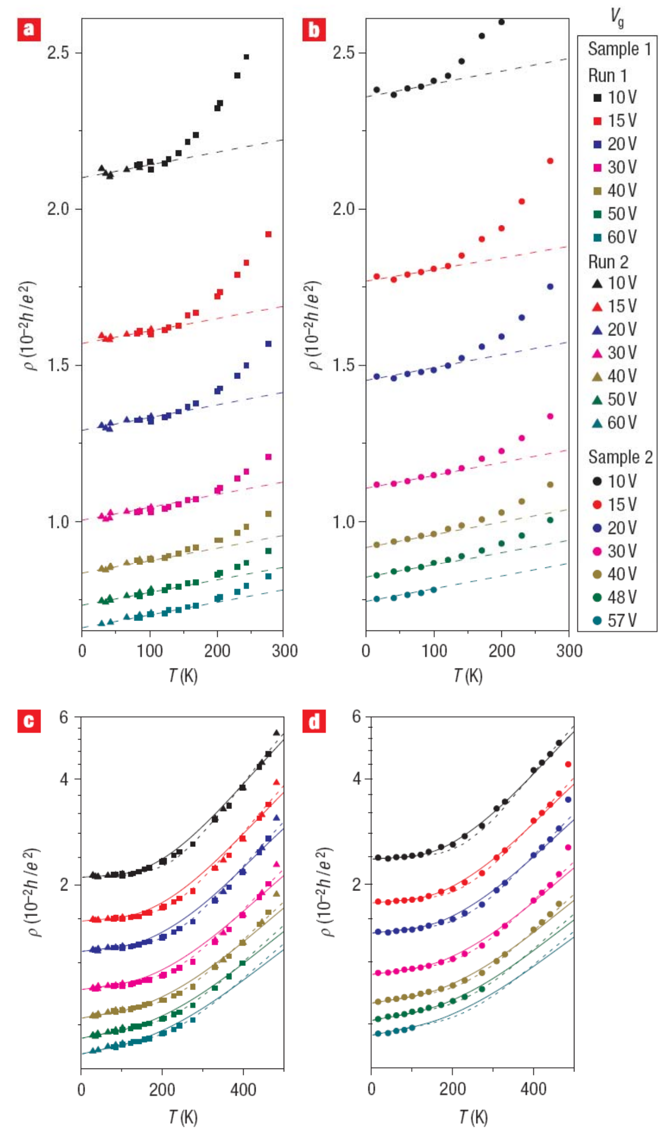
$$\rho(V_g, T) = \rho_0(V_g) + \rho_A(T); \quad \rho_A(T) = \left(\frac{h}{e^2}\right)\frac{\pi^2 D_A^2 k_B T}{2h^2 \rho_s v_s^2 v_F^2}$$

Measurements (see Fig. a,b)

- At higher T, strong deviation from linear T dependence
- Dependence also on Vg

→It suggests scattering on high energy phonon modes

Sample on SiO_2 substrate, UHV
T=[20K-500K], 4-point



Chen Nature Nanotech. 3, 206, (2008)

Scattering mechanisms in graphene

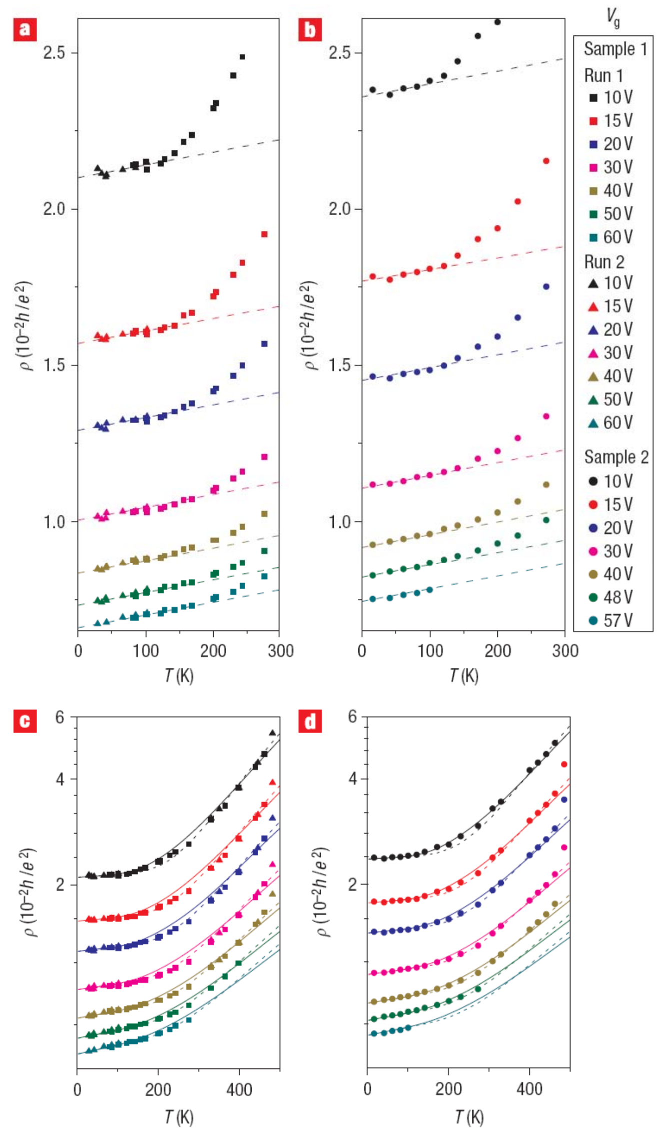
$$\rho(V_g, T) = \rho_0(V_g) + \rho_A(T) + \rho_B(V_g, T);$$

$$\rho_B(V_g, T) = B_1 V_g^{-\alpha_1}\left(\frac{1}{e^{(59\text{meV})/k_B T} - 1} + \frac{6.5}{e^{(155\text{meV})/k_B T} - 1}\right)$$

ρ_B: additional term to fit the measurements (see Fig. c,d)
Bose-Einstein distribution ~ population of high energy phonon modes, e.g. optical phonons
Very good fit of the measured curves with alfa=1.04

Optical phonons of graphene?
- Strong Vg dependence is not expected
- Mainly out of plane phonons at this energy. It is not expected to give strong contribution

Interfacial phonon scattering: Surface optical phonon modes in SiO_2 couples to e-s in graphene
The expected phonon energies and coupling strength (1:6.5) are inserted into **ρ_B**
Strong Vg dependence also expected



Scattering mechanisms in graphene

What limits the mobility at room T?

Different T dependence of ρ_0, ρ_A , ρ_B allows to separate the three contributions. ($\rho_B = \rho-\rho_0-\rho_A$)

Fig. a

ρ_A V_g independent

ρ_B ~V_g $^{i-1.04}$ relation confirmed

→ **Residual resistivity dominates**

Fig. b

Derive the mobility related to two e-p processes :

$\mu = 1/ne\rho = 1/c_g V_g e\, \rho$

→ SiO_2 contribution (c) dominates

→ **The intrinsic, LA phonon scattering mobility** at n= 10^{12}cm^{-2} (technologically relevant) : μ≈ 200 000 (see blue dot) **Higher than any known semiconductor!** (E.g. InSb ≈77 000 and carbon nanotubes ≈100 000).

Contributions at Room T

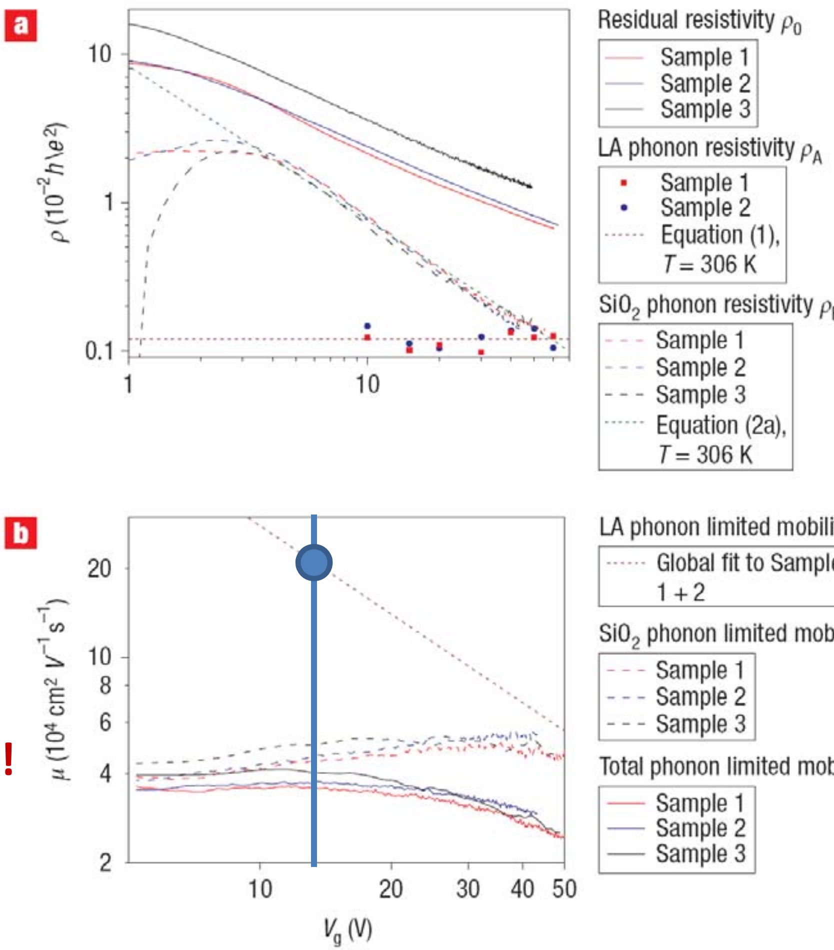


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Fig. c

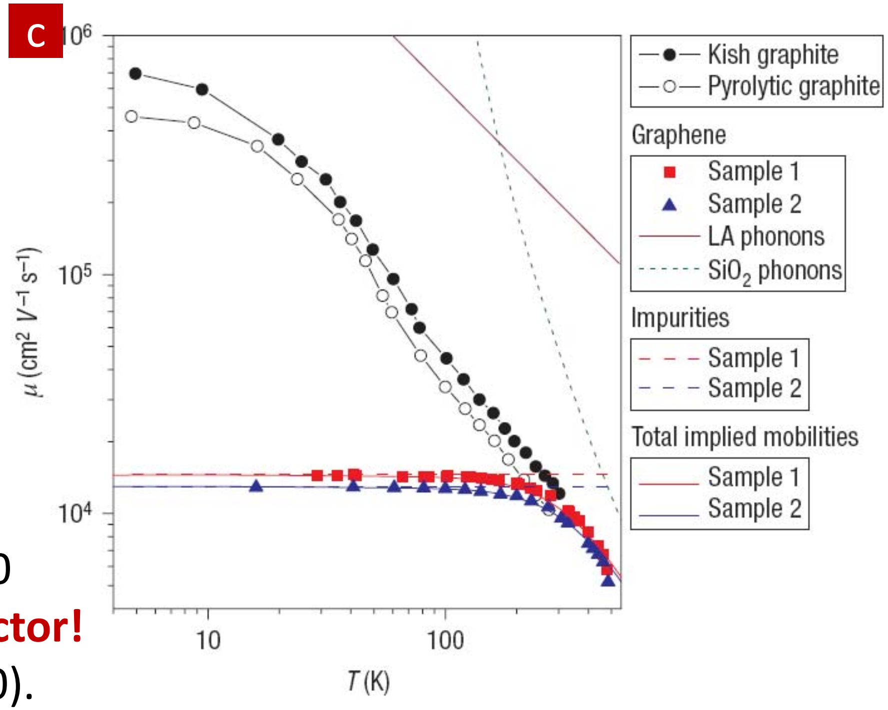
Comparision with graphites, sources of exfoliated graphene

Mobility is much smaller than for graphites. It is impurity dominated.

→Residual res. not due to point defects

but due to **charge impurities in SiO_2 substrate**

T dependence



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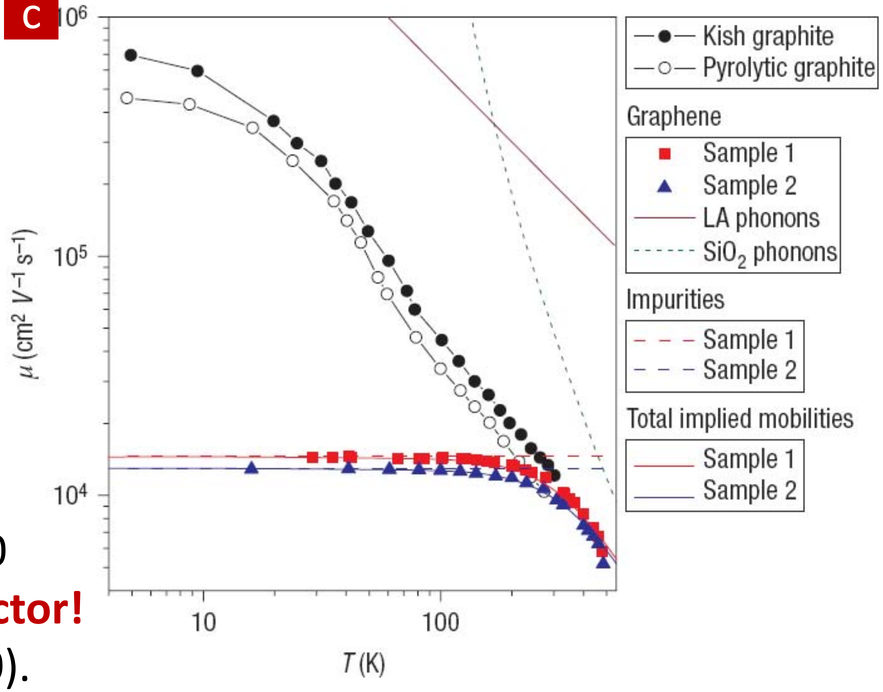
T dependence

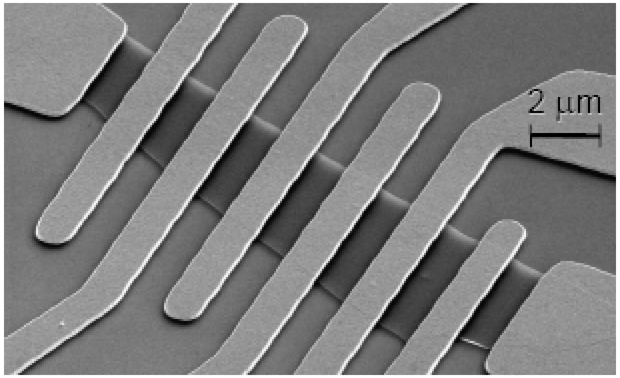
c



Σ: Problem, SiO_2 is bad substrate

Suspended flakes

SiO is etched by BHF



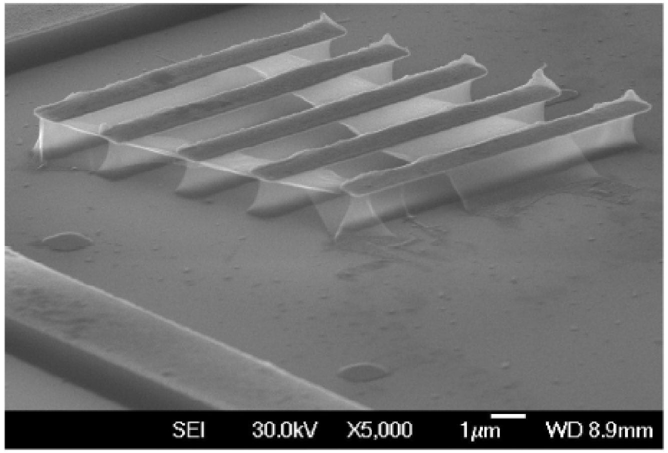
mobility up to 200,000cm^2/V·s
Mean free path L ~ µm

low-T mobilities
few million cm^2/V·s
Manchester, arxiv 2010

→ Demostration of FQHE

PMGI based organic polymer
N. Tombros arXiv:1009.4213

b



Possible with any metals ->
spin physics, supercondcutivity
600.000 cm2/Vs at n = 5.0 E9 cm-2 at 77K.
L~3µm

Suspended flakes

SiO is etched by BHF

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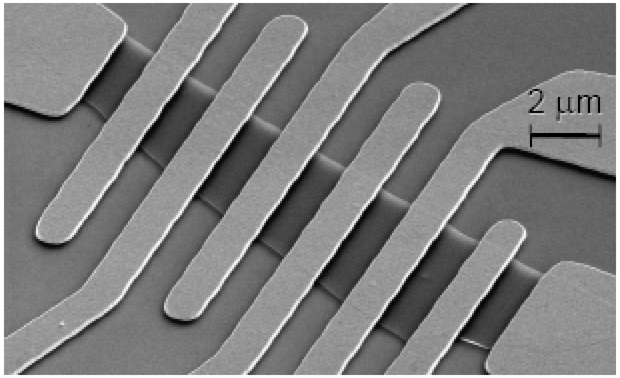
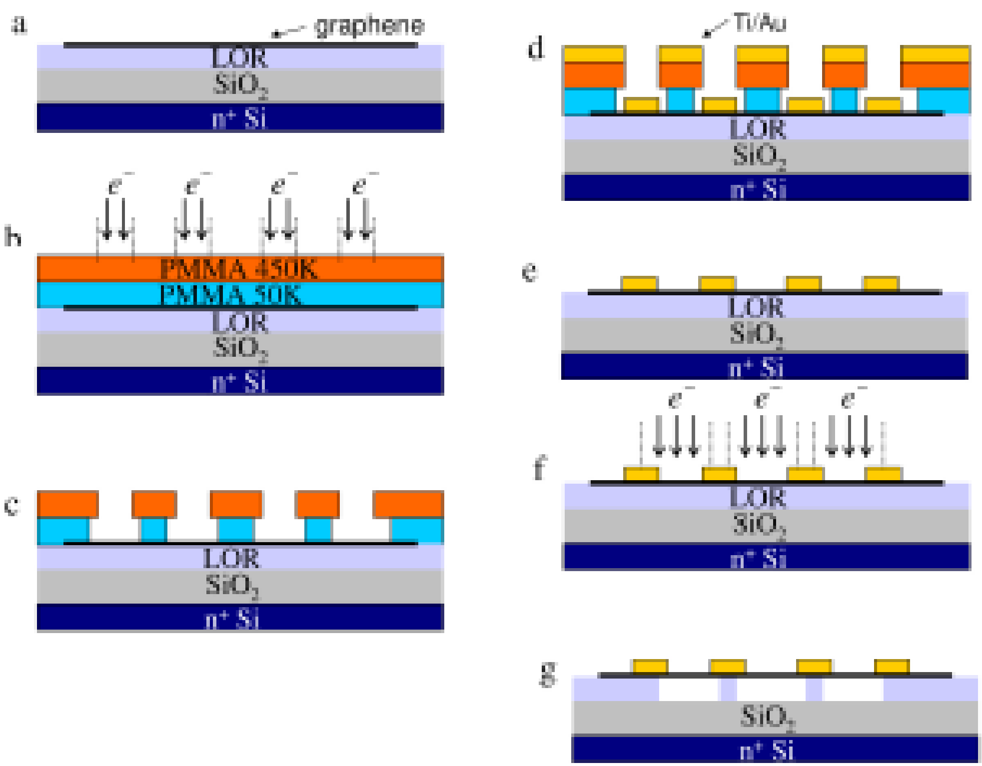


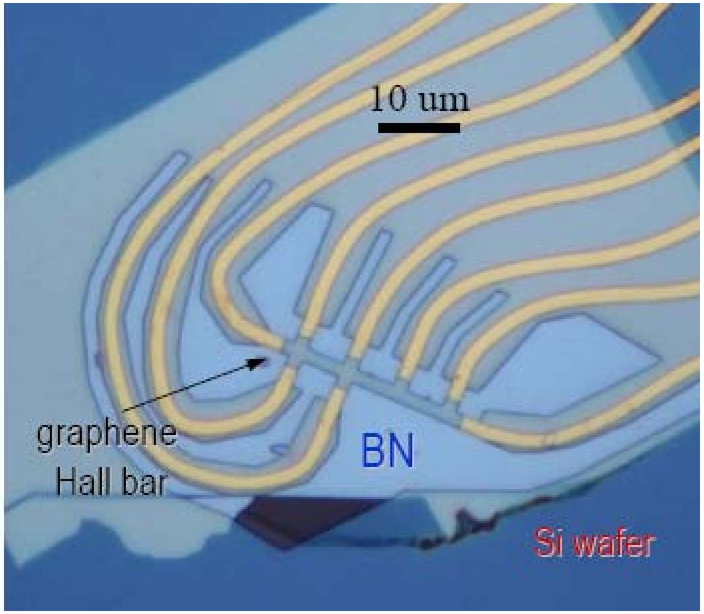


mobility up to 200,000cm^2/V·s
Mean free path L ~ μm

low-T mobilities
few million cm^2/V·s
Manchester, arxiv 2010

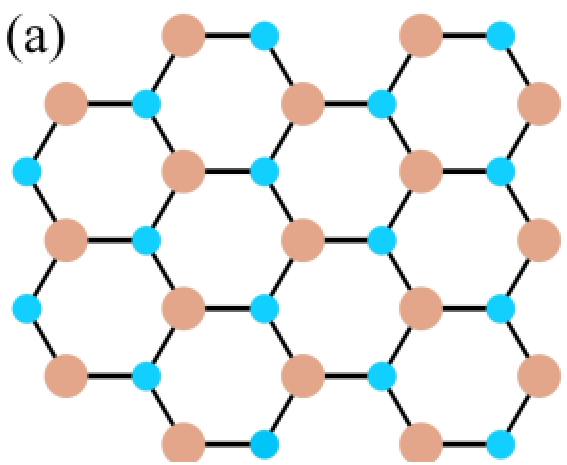
→ Demostration of FQHE

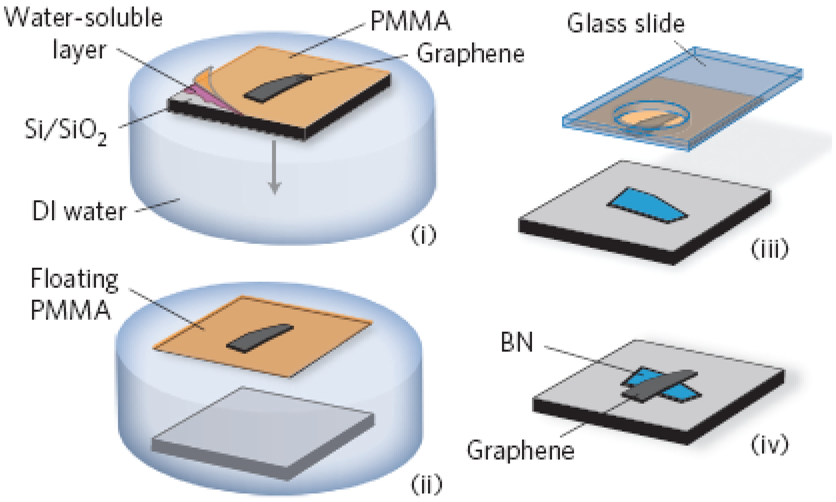
Better substrate – Boron Nitride



room-T mobility
close to 100,000 cm2/V·s

because it has an atomically smooth surface that is relatively free of dangling bonds and charge traps. It also has a lattice constant similar to that of graphite, and has large optical phonon modes and a large electrical bandgap.





Dean et. al., NatureNanotech 5, 722 (2010)

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Graphene based Hybrid Quantum Devices

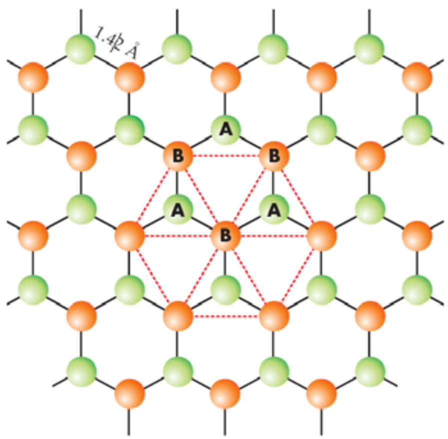
Sources:
http://www.tntconf.org/2010/Presentaciones/TNT2010_Geim.pdf

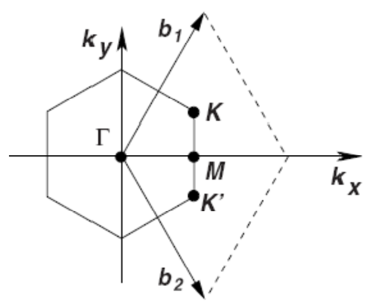
Band structure

Lattice, reciproce lattice

Unit cell with two atoms

A and B sublattice





Grafén tight binding sávszerkezeti leírása:

Első szomszéd hopping közelítésben (csak másik alrácsra ugorhat az elektron)

Hamilton 2*2 mátrix, A és B alrács komponensekre:

$$H = v_F\, \boldsymbol{\sigma} \bullet \mathbf{p}$$

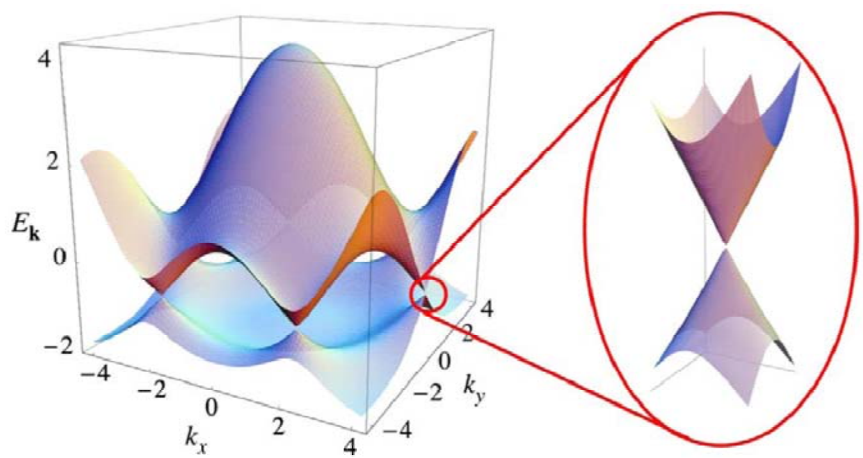
ahol σ a Pauli mátrixok, p az e. impulzusa, $v_F \approx 10^6$m/s

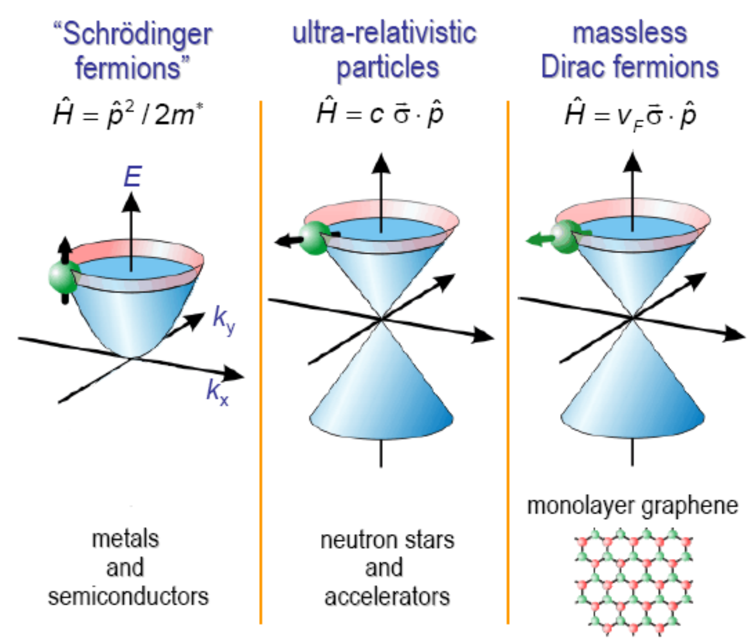
A két alrács pseudospinként viselkedik:

|↑> : A alrácson tartózkodás (zöld)

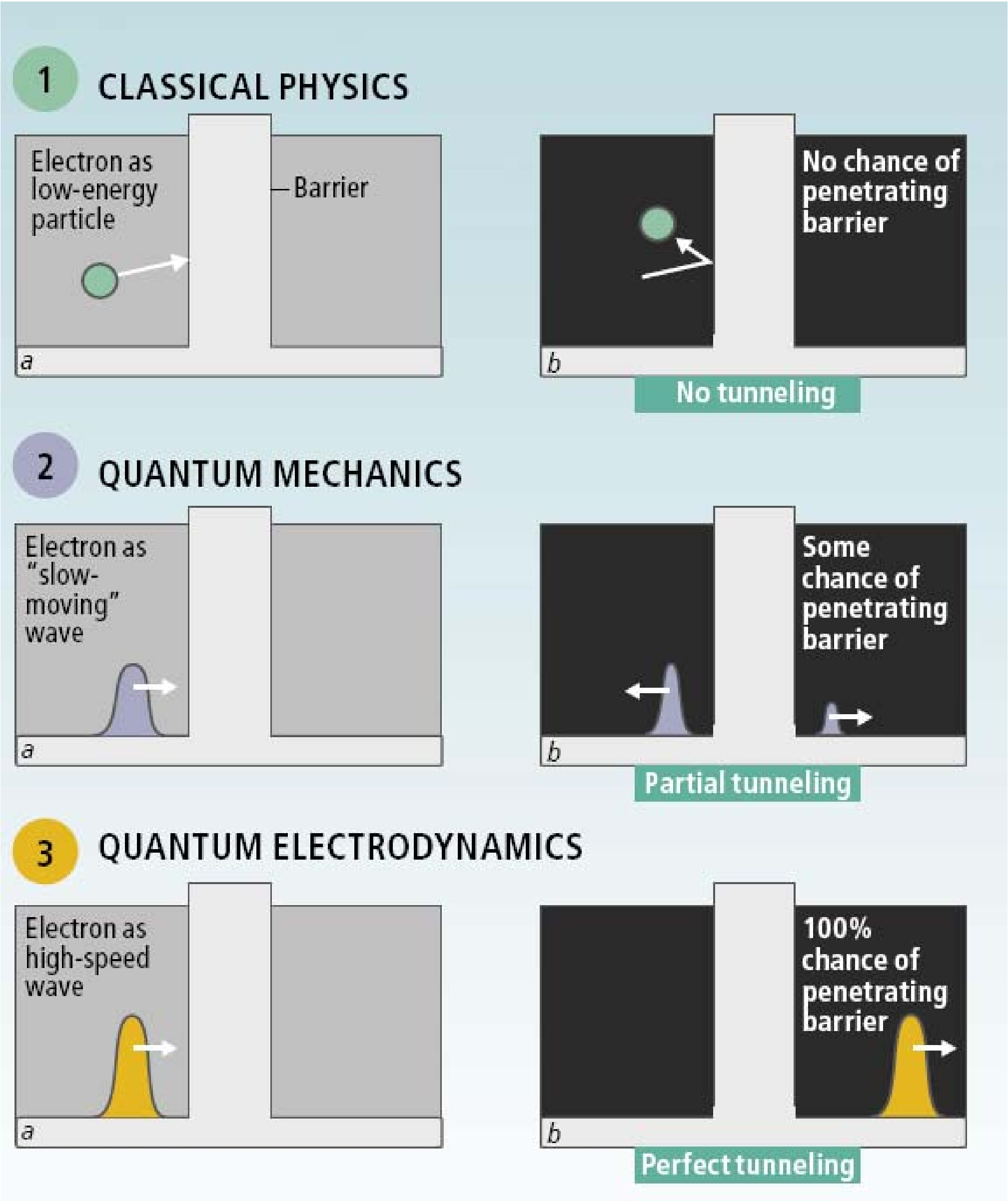
|↓> : B alrácson tartózkodás (piros)

→ Formailag a Dirac egyenlettel megegyező leírást, ahol spin szerepét átveszi a pszeudospin





Tunneling, Klein tunneling



Beenakker, Reviews of Modern Physics, 80, 1337 (2008)

Geim, Kim, Sci.Am. 298, 90 (2008)

Tunneling, Klein Backscattering

$$\frac{1}{m_{xx}} = \frac{v_F}{\hbar}\frac{k_y^2}{|k|^3}$$

Evolution of group velocity:

$$\frac{dv_x}{dt} \equiv \frac{1}{m_{xx}}F_x = \frac{1}{m_{xx}}(-e)E_0 \qquad (*)$$

In linear electrostatic potential:

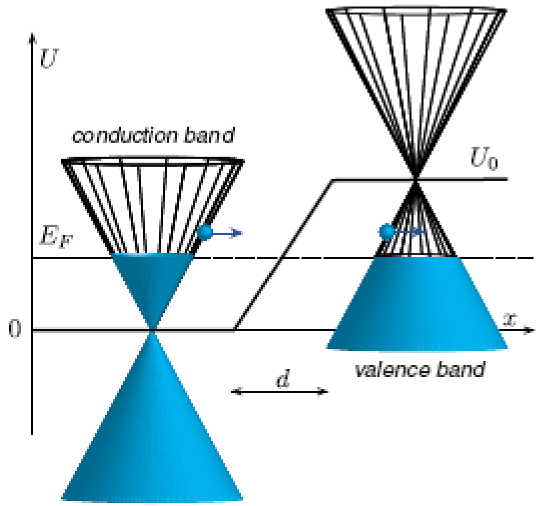
$$V = E_0 x, \quad E_x = E_0, \quad F_x = -eE_0$$

At normal incident: $k_y = 0 \rightarrow \frac{dv_x}{dt} = 0 \rightarrow$ backscattering is avoided

Electron can propagate through an infinite high potential barrier.

N-P junction:

Potential profile with a step of U_0 at a distance d



Beenakker, Reviews of Modern Physics, 80, 1337 (2008)

Klein scattering:

perfect transmission at normal incident

Tunneling, Klein tunneling

More precisely, quasi classical dynamics

Two Dirac cones: Conduction band $E = \hbar v_F |k|$,

Valance band $E = -\hbar v_F |k|$

$$\vec{v} \equiv \frac{1}{\hbar}\frac{\partial E}{\partial \vec{k}} = \frac{1}{\hbar}\hbar v_F \frac{\vec{k}}{|k|} = v_F \overrightarrow{e_k} = v_F^2 \frac{\vec{k}}{E},$$

thus $|v| = v_F$, $\vec{v}||\vec{k}$

$$\hbar\dot{\vec{k}} \equiv \vec{F} = -eE_0\overrightarrow{e_x}$$

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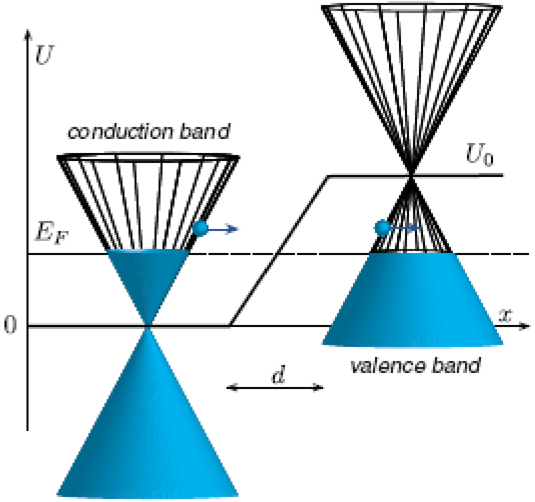


Beenakker, Reviews of Modern Physics, 80, 1337 (2008)

Effect of the potential profile, U (see figure):

- k decreases and changes sign
- based on (*), $\vec{v}$ stays constant, i.e. $\vec{v} = v_F\overrightarrow{e_x}$.

→ e ends up in the valence band

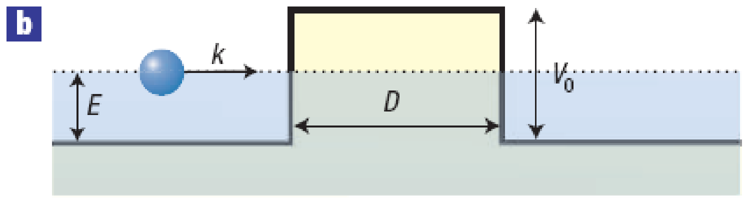
Klein scattering:

perfect transmission at normal incident

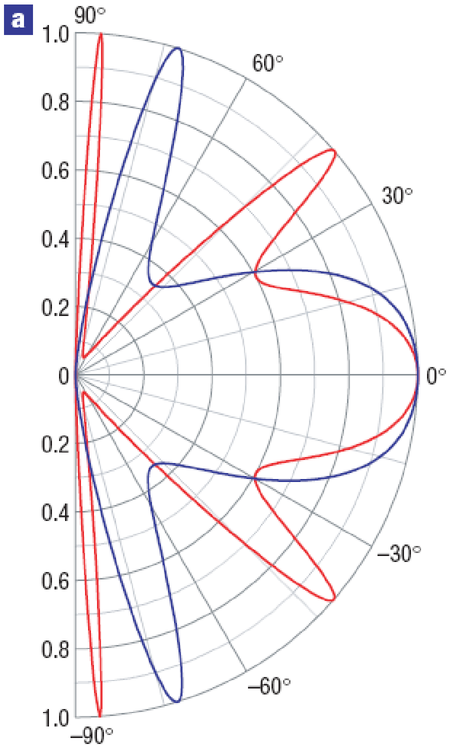
Tunneling, Klein tunneling

Result of proper calculation

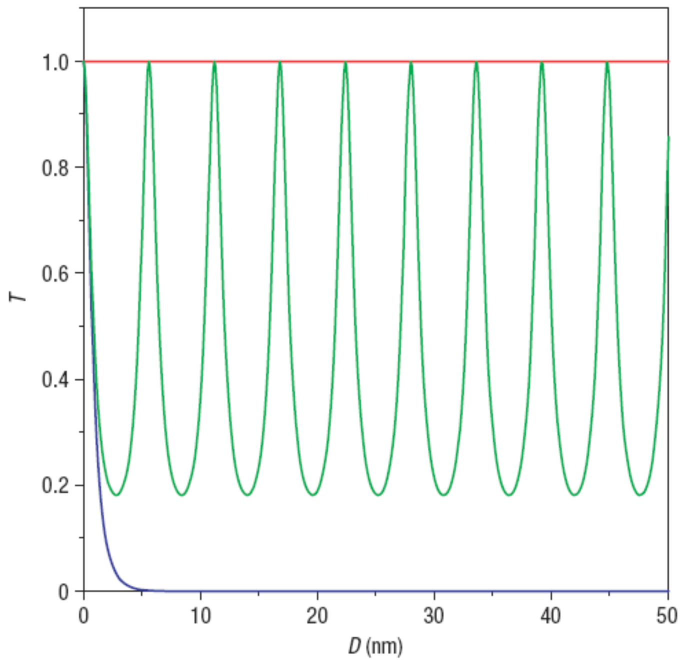
Wave function matching



Transmission probability vs. D
of normally incident electrons
- in single- and bi-layer graphene (red and blue curves,respectively) and in a non-chiral zero-gap semiconductor (green curve)



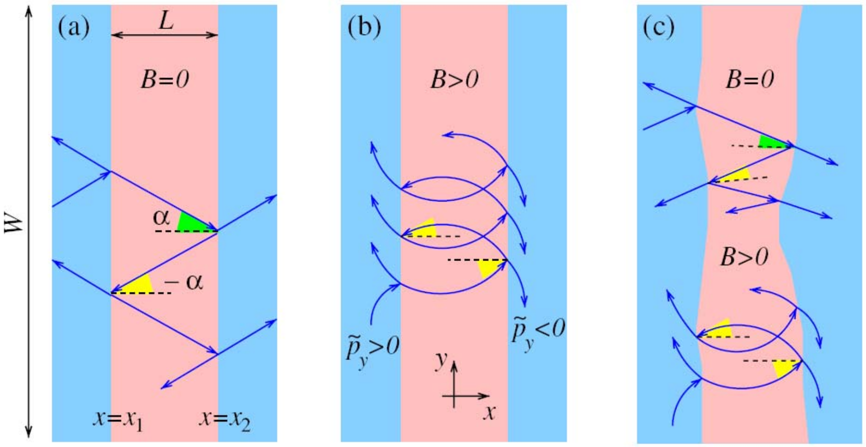
Transmission probability T
through a 100-nm-wide barrier as a function of the incident angle, two different barrier height



→ Difficult to measure since e-s out of normal incident also arrive

Katsnelson et al Nature Physics, 2, 620 (2006)

Klein backscattering & Fabry-Perot Interferences



Backscattering on P-N-P junction

When incident angle, α is varied from positive to negative, phase of the reflection amplitude (R) jumps π. Its sign changes. (At α=0, R=0).

If α<>0 → R>0, several scatterings in P-N- P → interference pattern

Accumulated phase in one circle:

$$\Delta\theta = 2\theta_{WBK} + \Delta\theta_1 + \Delta\theta_2$$

where θ_{WBK} phase from travelling in N

$\Delta\theta_1, \Delta\theta_2$ Klein backreflection phase of the interfaces

At B=0 (see Fig. a) the incident angles $\Delta\theta_{1(2)}$ at P-N and N-P have opposite signs → jumps in $\Delta\theta_1$, $\Delta\theta_2$ cancels

At B>0 (see Fig. b), trajectories are curved, → incident angles at P-N and N-P can be equal

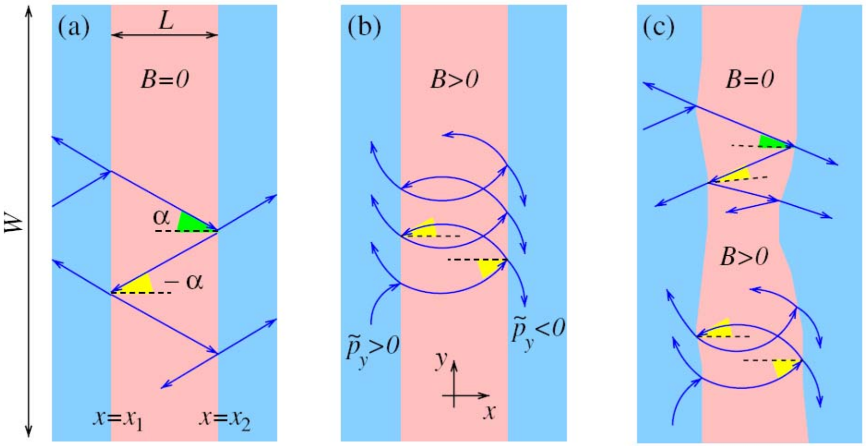
In this case one can show that $\Delta\theta_1 + \Delta\theta_2 = \pi$ (It is a Barry phase!)

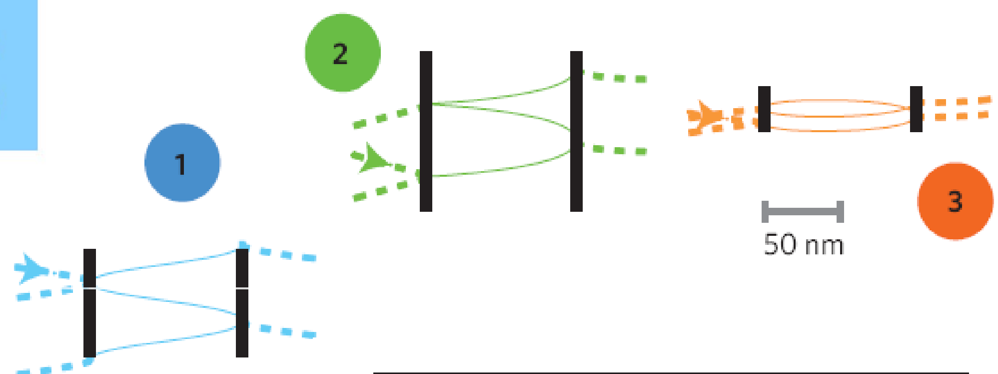
Thus for B=0 ↗ and trajectories with small p_y π shift is expected (i.e. sign change) transmission amplitude

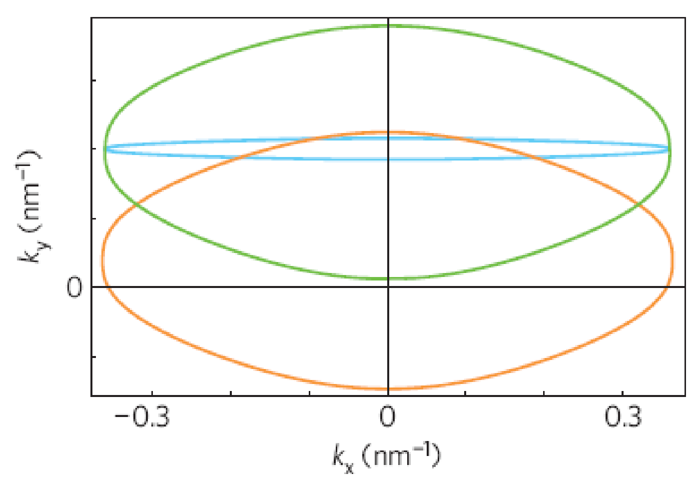
(Fig.c) one can show, it is robust against barrier roughness

Shytov et al. PRL 101, 156804 (2008)

Klein backscattering & Fabry-Perot Interferences







Remark (Barry-phase):

Trajectory in Fig.a corresponds to **1**
Trajectory in Fig.b corresponds to **3**
The main difference that during one circle between P-N and N-P:
the k vactor of **3** goes around k=0 while for **1** NOT.
This generates the Barry phase:
Due to the chiral symmetry, topological singularity at degenerecy point of the band structure k=0.

Shytov et al. PRL 101, 156804 (2008)

Klein backscattering & Fabry-Perot Interferences

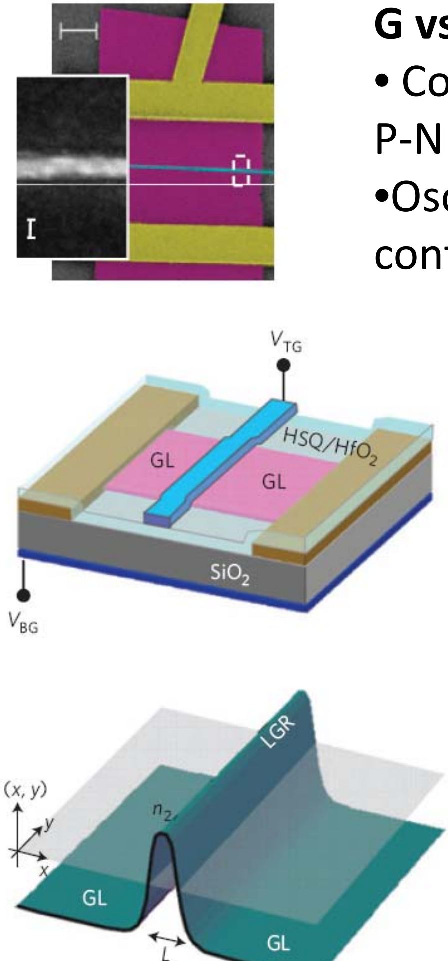
N-P-N device

Separate gating by backgate and topgate

Topgate width=20nm! → ballistic

a

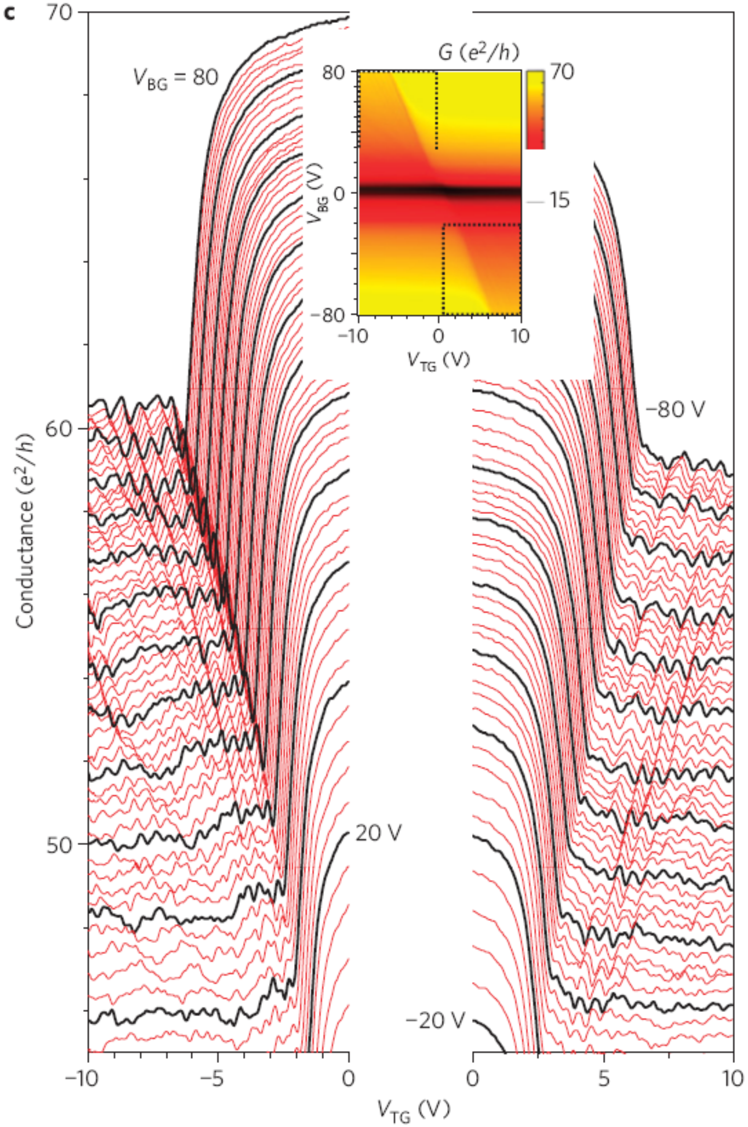
b



G vs. V_{TG} vs. V_{BG}

- Conductance is lower when N-P-N setting instead of N-N-N
- Oscillations at N-P-N configuration:
  - V_{TG} varies pot. barrier → θ_{WBK} → oscillations
  - Oscillatory G is induced by trajectories with incident angle where neither T, nor R is large (i.e. α not too small)

c

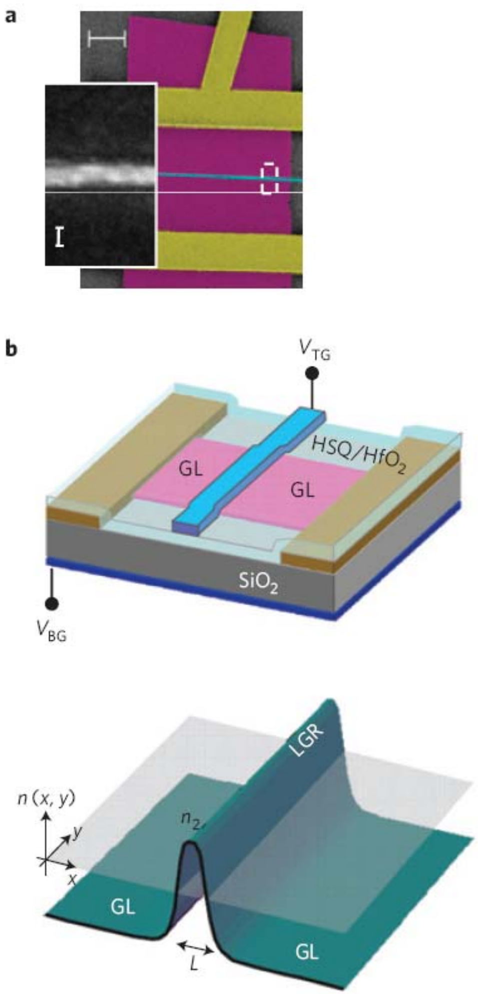


Young et al. Nature Physics 5, 222 (2009)

Klein backscattering & Fabry-Perot Interferences

N-P-N device

Separate gating by backgate and topgate
Topgate width=20nm! → ballistic

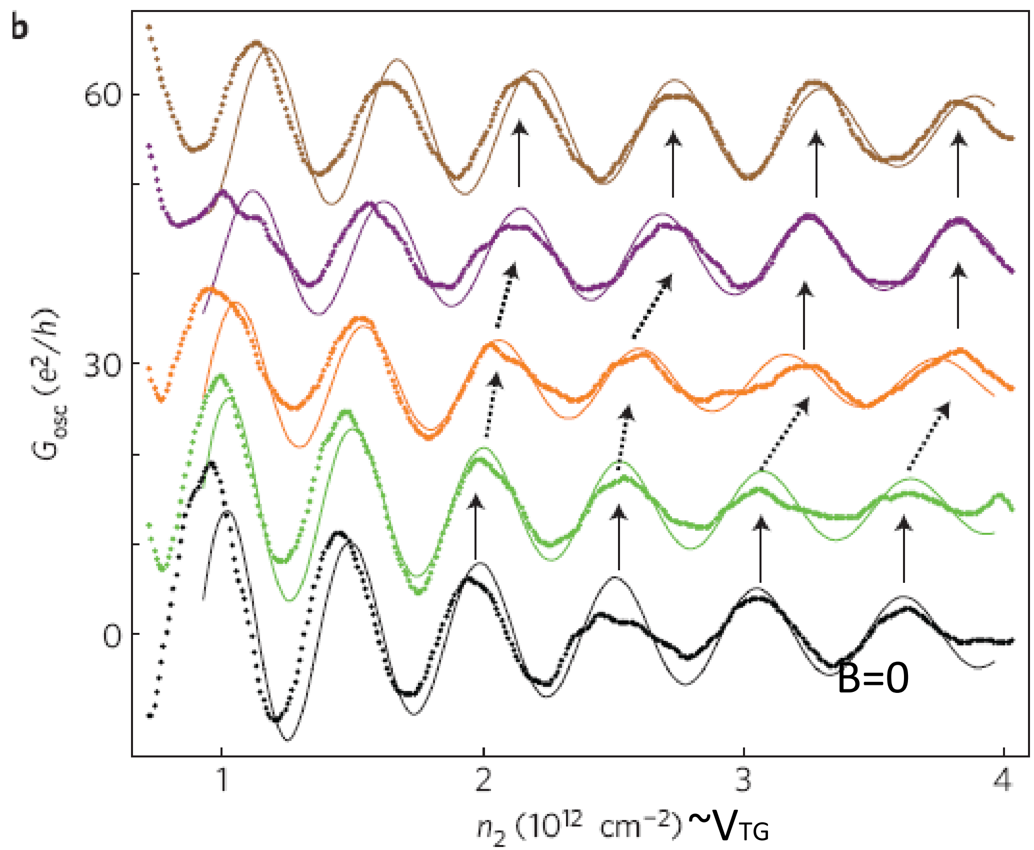


Young et al. Nature Physics 5, 222 (2009)

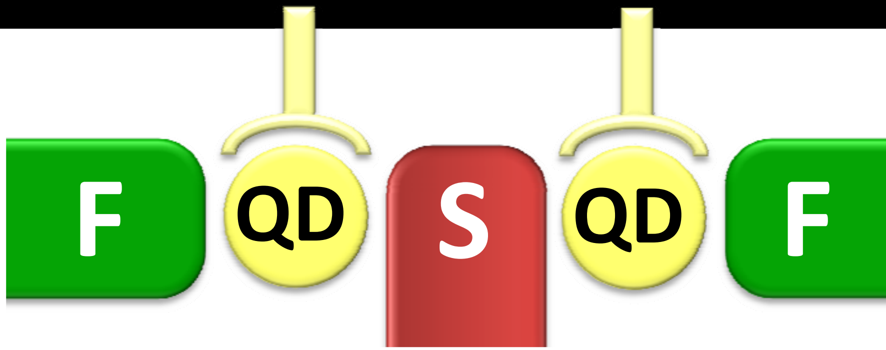
G oscillations vs. B *(Dots experiment, line theory)*

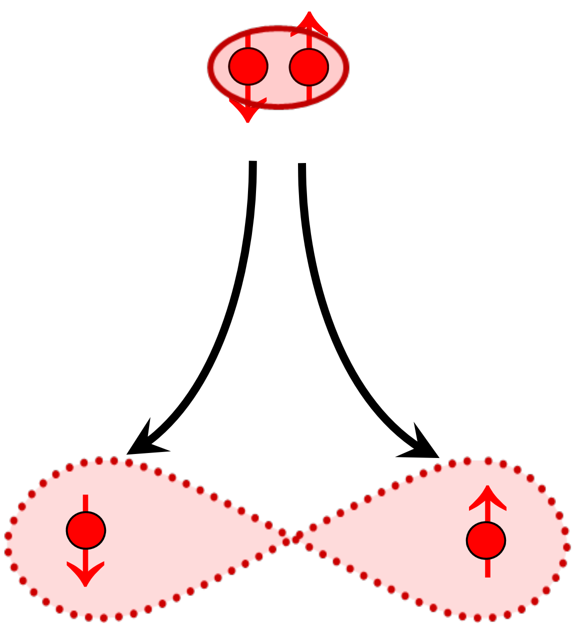
At different B fields (B=0, 200, 400, 600, 800mT) the oscillations of G.

In this B range ≈ π shift is induced in the interference patern.

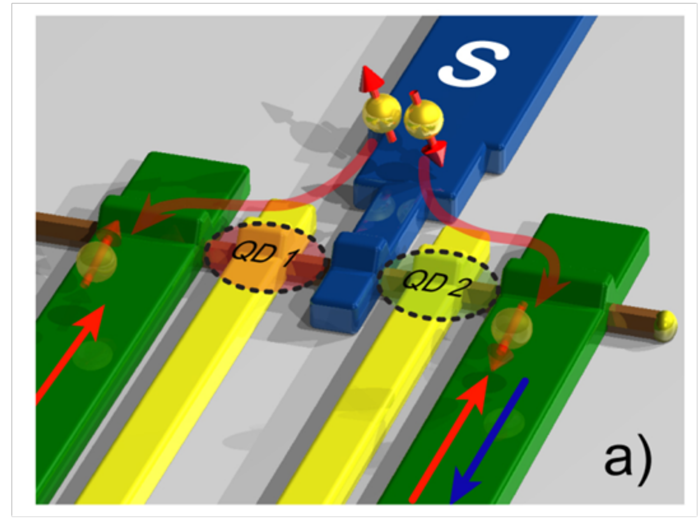


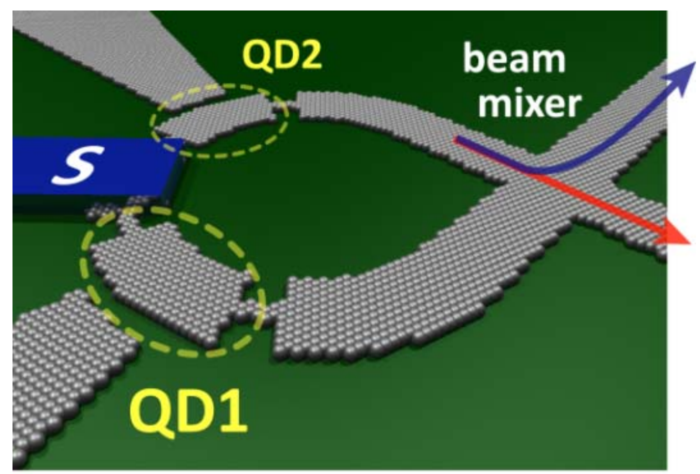
Hybrid Graphene Devices



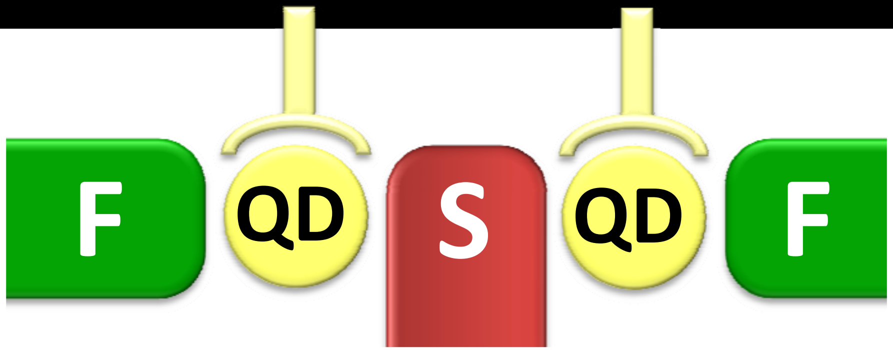


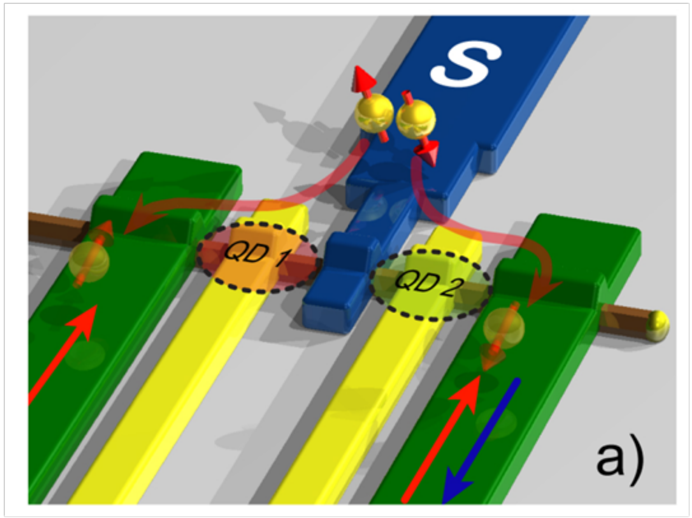
Entanglement?

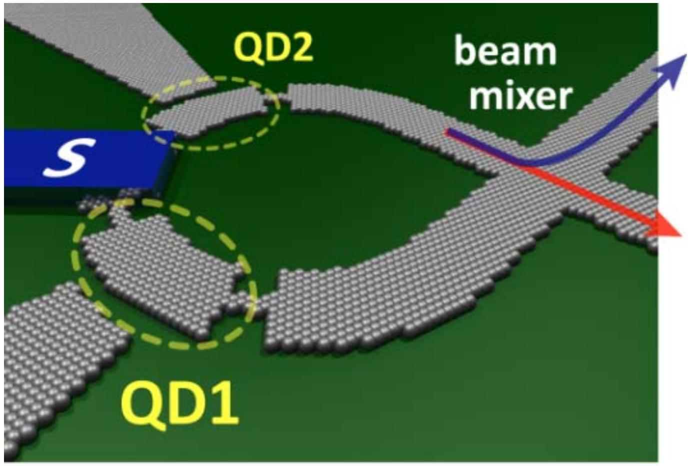


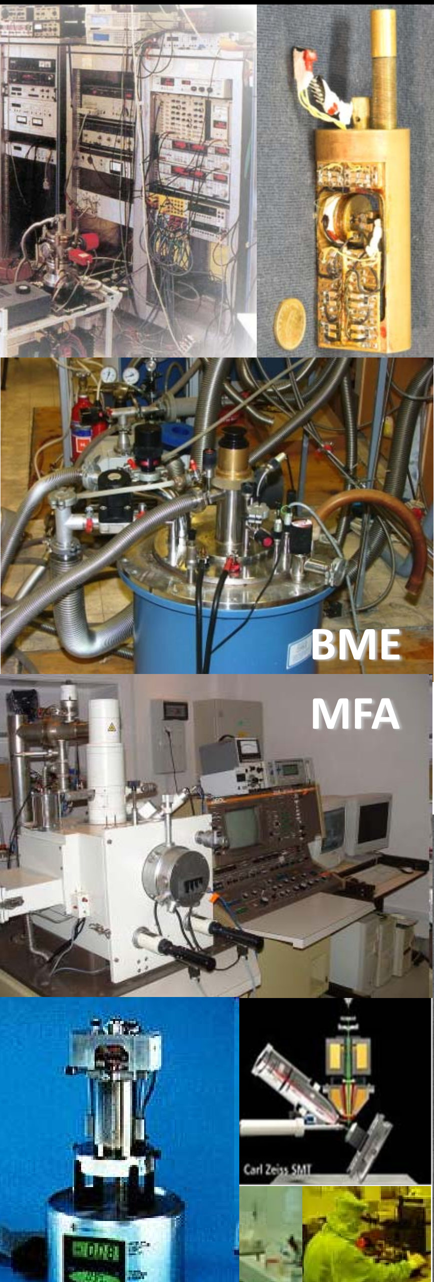


Hybrid Graphene Devices









Infrastructure

BME:

Low T transport lab: He liquefier, 4 cryostats, He4, He3 systems, electronics, MCBJ, Kerr …

BME & MFA Joint lab:

E-lithography: JEOL 848 + Raith Elphy; LeoXBeam SEM/FIB, AFM, STM, Clean room (300m^2), Raman, …

Collaboration with L.P. Biro MFA