Supersymmetry Complement to Dark Matter Particle Search at Accelerators

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Contains summaries of

- Supersymmetry for variables defined at a single space point or supersymmetric quantum mechanics
- Superfield and Wess-Zumino action
- MSSM

Adding Grassmann-variables $\Theta, \overline{\Theta}$ to t:

$$\overline{\Theta}\Theta + \Theta\overline{\Theta} = 0, \qquad \Theta^2 = \overline{\Theta}^2 = 0, \qquad f(\Theta, t) = f_0(t) + f_1(t)\Theta \tag{1}$$

Operational rules for Θ (Berezin):

$$\frac{d}{d\Theta_i}1 = 0, \quad \frac{d\Theta_i}{d\Theta_j} = \delta_{ij}, \quad \frac{d}{d\Theta_i}(\Theta_k\Theta_l) = \delta_{ik}\Theta_l - \delta_{il}\Theta_k,$$
$$\int d\Theta_i 1 = 0, \quad \int d\Theta_i\Theta_j = \delta_{ij}. \tag{2}$$

Super degree of freedom:

$$\Phi(t, \Theta, \overline{\Theta}) = \phi(t) + \Theta\overline{\psi}(t) + \psi(t)\overline{\Theta} + D(t)\Theta\overline{\Theta}. \tag{3}$$

Supersymmetry transformation ("parametrized" by Grassmann variables $\zeta, \overline{\zeta}$):

$$t \to t + i\Theta\overline{\zeta} - i\zeta\overline{\Theta}, \quad \Theta \to \Theta + \zeta, \quad \overline{\Theta} \to \overline{\Theta} + \overline{\zeta} \tag{4}$$

Transformation of superfield components:

$$\phi(t) \to \phi(t) + \zeta\overline{\psi} + \psi\overline{\zeta}, \qquad \psi(t) \to \psi(t) - i\zeta\frac{d\phi}{dt} + \zeta D(t),$$

$$D(t) \to D(t) - i\frac{d\psi}{dt}\overline{\zeta} + i\zeta\frac{d\overline{\psi}}{dt} = i\frac{d}{dt}(\zeta\overline{\psi} - \psi\overline{\zeta}). \tag{5}$$

Important note: Variation of $D(t)$ is a complete time-derivative!

Chiral (Antichiral) fields: depend only on Θ ($\overline{\Theta}$)

Covariant (Anti-covariant) derivative:

$$\mathcal{D} = \frac{\partial}{\partial\Theta} + i\overline{\Theta}\frac{\partial}{\partial t}, \qquad \overline{\mathcal{D}} = -\left(\frac{\partial}{\partial\overline{\Theta}} + i\Theta\frac{\partial}{\partial t}\right)$$

$\mathcal{D}\Phi(t,\Theta)$ is anti chiral, $\overline{\mathcal{D}}\Phi(t,\Theta)$ is chiral
and depend on $t - i\Theta\overline{\Theta}$ and $t + i\Theta\overline{\Theta}$, respectively.
Supersinglet projection with Berezin integration:

$$S_{SUSY} = \int dt d\Theta d\overline{\Theta} \left[\frac{1}{2}\overline{\mathcal{D}}\Phi\mathcal{D}\Phi - F[\Phi]\right]. \tag{6}$$

Combined integration over $\Theta, \overline{\Theta}$ "projects out" the D-term. Its supervariation is a full derivative which leads to the super-invariance of the action:

$$\begin{aligned}
\frac{1}{2}\int dt d\Theta d\overline{\Theta}\left(\overline{\mathcal{D}}\Phi\mathcal{D}\Phi\right) &= \int dt\left[\frac{1}{2}\left(\frac{d\phi}{dt}\right)^2 + \frac{1}{2}D^2 + i\frac{d\psi}{dt}\overline{\psi}\right], \\
\int dt d\Theta d\overline{\Theta} F[\Phi] &= \int dt\left(F'[\phi]D + F''[\phi]\overline{\psi}\psi\right).
\end{aligned} \tag{7}$$

The component D has no intrinsic dynamics: $D = F'$, easy to eliminate

$$L_{SUSY} = \frac{1}{2}\left(\frac{d\phi}{dt}\right)^2 - \frac{1}{2}[F'(\phi)]^2 - i\overline{\psi}\frac{d\psi}{dt} - F''(\phi)\overline{\psi}\psi$$

Generalisation to fields living in Minkowski space:

Grassmann variables with spinor index: $\Theta_\alpha, \overline{\Theta}_{\dot\alpha}, \quad \alpha, \dot\alpha = 1, 2$

Completion of Pauli matrices to four Lorentz indices:

$$\sigma^\mu_{\alpha\dot\alpha} = (I_{\alpha\dot\alpha}, \sigma_{\alpha\dot\alpha}), \quad \overline{\sigma}^\mu_{\alpha\dot\alpha} = (I_{\alpha\dot\alpha}, -\sigma_{\alpha\dot\alpha})$$

Super-transformation:

$$(x^\mu, \Theta, \overline{\Theta}) \to (x^\mu + a^\mu - i\xi\sigma^\mu\overline{\Theta} + i\Theta\sigma^\mu\bar\xi, \Theta + \xi, \overline{\Theta} + \bar\xi)$$

Super-generators and superalgebra:

$$P_\mu = i\partial_\mu, \quad iQ_\alpha = \frac{\partial}{\partial\Theta^\alpha} - i\sigma^\mu_{\alpha\dot\alpha}\overline{\Theta}^{\dot\alpha}\partial_\mu, \quad i\overline{Q}_{\dot\alpha} = \frac{\partial}{\partial\overline{\Theta}^{\dot\alpha}} - i\Theta^\alpha\overline{\sigma}^\mu_{\alpha\dot\alpha}\partial_\mu \qquad (8)$$

$$[Q_\alpha, \overline{Q}_{\dot\alpha}]_+ = 2\sigma^\mu_{\alpha\dot\alpha}P_\mu, \qquad P_\mu = \frac{1}{2}(\overline{\sigma}^\mu)^{\alpha\dot\alpha}[Q_\alpha, \overline{Q}_{\dot\alpha}]_+ \qquad (9)$$

Covariant derivatives:

$$D = \frac{\partial}{\partial\Theta} + i\sigma^\mu\overline{\Theta}\partial_\mu, \quad \overline{D} = -\frac{\partial}{\partial\overline{\Theta}} - i\Theta\sigma^\mu\partial_\mu, \qquad [D_\alpha, \overline{D}_{\dot\alpha}] = 2i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu$$

Superfields General:

$$S(x, \Theta\overline{\Theta}) = f(x) + \Theta\varphi(x) + \overline{\theta}\bar{\chi}(x) + \Theta\Theta m(x) + \overline{\Theta}\overline{\Theta}n(x) + \Theta\sigma^\mu\overline{\Theta}V_\mu(x)$$

$$+ \Theta\Theta\overline{\Theta}\bar{\lambda}(x) + \overline{\Theta}\overline{\Theta}\Theta\psi(x) + \Theta\Theta\overline{\Theta}\overline{\Theta}d(x)$$

Chiral: $\overline{D}\Phi(x) = 0$ Anti-chiral: $D\Phi^* = 0$

$$\begin{aligned}\Phi(x, \Theta, \overline{\Theta}) \quad &= \quad \varphi(x) + \sqrt{2}\Theta\psi(x) + \Theta\Theta F(x) + i\partial_\mu\varphi(x)\Theta\sigma^\mu\Theta \\ &\quad -\frac{i}{\sqrt{2}}\Theta\Theta\partial_\mu\psi\sigma^\mu\overline{\Theta} - \frac{1}{4}\partial_\mu\partial^\mu\varphi(x)\Theta\Theta\overline{\Theta}\overline{\Theta}\end{aligned} \qquad (10)$$

Super-variation of F (the coefficient of $\Theta\Theta$) is full derivative:

$$\delta F = \frac{1}{2}\partial_\mu(\psi(x)\sigma^\mu\bar{\zeta})$$

Vector-field:

$$S(x,\Theta,\overline{\Theta}) = S^*(x,\Theta,\overline{\Theta})$$

Example: $\Phi^*\Phi$

$$V(x,\Theta,\overline{\Theta}) = C(x)+i\Theta\chi(x)-i\overline{\Theta}\bar{\chi}(x)+\frac{i}{2}\Theta\Theta(M(x)+iN(x))-\frac{i}{2}\overline{\Theta\Theta}(M(x)-iN(x))$$

$$+\Theta\sigma^{\mu}\overline{\Theta}V_{\mu}+i\Theta\Theta\overline{\Theta}(\bar{\lambda}(x)+\frac{i}{2}\overline{\sigma}^{\mu}\partial_{\mu}\chi(x))-i\overline{\Theta\Theta}\Theta(\lambda(x)+\frac{i}{2}\sigma^{\mu}\partial_{\mu}\chi(x))$$

$$+\frac{1}{2}\Theta\Theta\overline{\Theta\Theta}\left(D-\frac{1}{2}\partial_{\mu}\partial^{\mu}C(x)\right)$$

The variation of the coefficient of $\Theta\Theta\overline{\Theta\Theta}$ is full derivative (D-term)

Lagrangian (Generalisation of the Wess-Zumino model, 1974)

$$L = \sum_i \int d^2\Theta \int d^2\overline{\Theta}\Phi^*\Phi + \left(\int d^2\Theta W[\Phi] + \text{c.c.}\right)$$

$$W[\Phi] = \frac{1}{2}m_{ij}\Phi\Phi + \frac{1}{3}\lambda_{ijk}\Phi_i\Phi_j\Phi_k \tag{11}$$

In terms of the component fields:

$$L = \partial_\mu\varphi_i^*\partial^\mu\varphi_i + i\bar{\psi}_i\overline{\sigma}^\mu\partial_\mu\psi_i$$
$$+F_i^*F_i + [m_{ij}(\varphi_iF_j - \tfrac{1}{2}\psi_i\psi_j) + \lambda_{ijk}(\varphi_i\varphi_jF_k - \psi_i\psi_j\psi_k) + \text{c.c.}] \tag{12}$$

Excluding the non-dynamical $F_i(x)$ fields:

$$L = \partial_\mu\varphi_i^*\partial^\mu\varphi_i + i\bar{\psi}_i\overline{\sigma}^\mu\partial_\mu\psi_i - |m_{ij}\varphi_j + \lambda_{ijk}\varphi_j\varphi_k|^2 - [\frac{1}{2}m_{ij}\psi_i\psi_j + \lambda_{ijk}\psi_i\psi_j\psi_k + \text{c.c.}] \tag{13}$$

Minimal Supersymmetric Standard Model (MSSM)
Superfield extensions of the SM-fields:

- Q_L, balkezes kvark, $SU(2)_W$ dublett, $SU(3)_S$ triplett, $Y_Q = 1/6$,
- u_R, jobbkezes anti-kvark, $SU(2)_W$ szinglett, $SU(3)_S$ triplett, $Y_u = -2/3$,
- d_R, jobbkezes anti-kvark, $SU(2)_W$ szinglett, $SU(3)_S$ triplett, $Y_d = 1/3$,
- L_L, balkezes lepton, $SU(2)_W$ dublett, $SU(3)_S$ szinlett, $Y_L = -1/2$,
- e_R, $SU(2)_W$ szinglett, $SU(3)_S$ szinglett, $Y_e = 1$,
- H_1, H_2, Higgs multiplettek, $SU(2)_W$ dublett, $SU(3)_S$ szinglett, $Y_1 = -Y_2 = 1$,
- W, $SU(2)_W$ triplett, $SU(3)_S$ szinglett, $Y_W = 0$,
- B, $SU(2)_W$ szinglett, $SU(3)_S$ szinglett, $Y_B = 2$,
- G, $SU(2)_W$ szinglett, $SU(3)_S$ oktett, $Y_G = 0$.

Yukawa-type superpotential:

$$W = -\mu H_1 H_2 + g_e H_{1\alpha} L_{L\beta} \epsilon_{\alpha\beta} e_R + g_d H_{1\alpha} Q_{L\beta} \epsilon_{\alpha\beta} d_R + g_u H_{2\alpha} Q_{L\beta} \epsilon_{\alpha\beta} u_R. \quad (14)$$

Eliminating auxiliary fields:

$$\begin{aligned} V_{pot}^{(1)} &= |g_u H_2 u_R + g_d H_1 d_R|^2 + |-\mu H_1 + g_u Q_L u_R|^2 \\ &+ |-\mu H_2 + g_e L_L e_R + g_d Q_L d_R|^2 \\ &+ g_d^2 |\tilde{H}_1^* Q_L|^2 + g_u^2 |\tilde{H}_2^* Q_L|^2 + g_l^2 |\tilde{H}_1^* L_L|^2. \end{aligned} \quad (15)$$

Further scalar-fermion interaction from the vector-superfields, needed for the gauge interaction:

$$\begin{aligned} V_{pot}^{(2)} &= \frac{1}{2} g_W^2 \left(H_1^\dagger t_W^a H_1 + H_2^\dagger t_W^a H_2 + Q_L^\dagger t_W^a Q_L + L_L^\dagger t_W^a L_L \right)^2 \\ &+ \frac{1}{2} g_S^2 \left(Q_L^\dagger t_S^a Q_L + u_R^\dagger t_S^a u_R + d_R^\dagger t_S^a d_R \right)^2 . \end{aligned} \quad (16)$$

Higgs symmetry breaking:

$$< H_1 >= \begin{pmatrix} 0 \\ H_{10} \end{pmatrix}, \qquad \langle H_2 \rangle = \begin{pmatrix} H_{20} \\ 0 \end{pmatrix}, \qquad \tan\beta \equiv \frac{H_{20}}{H_{10}} \tag{17}$$

The coupled neutralino sector for the Lightest Supersymmetric Particle (LSP):

$$M_{neut} = \begin{pmatrix} M_1 & 0 & -M_Z c_b s_W & M_Z s_b s_W \\ 0 & M_2 & M_Z c_b c_W & M_Z s_b c_W \\ -M_Z c_b s_W & M_Z c_b c_W & 0 & -\mu \\ M_Z s_b s_W & -M_Z s_b c_W & -\mu & 0 \end{pmatrix}. \tag{18}$$

$s_W = \sin\Theta_W, c_W = \cos\Theta_W, s_b = \sin\beta, c_b = \cos\beta$