Cosmology Complement to Dark Matter Particle Search at Accelerators

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Contains summaries of

- Einstein equations in FRW geometry
- Boltzmann equation in expanding Universe

Friedman-Robertson-Walker space-time geometry:

$$ds^2 = c^2 dt^2 - a(t)^2 \left[\frac{dr^2}{1 + kr^2} + r^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2) \right]. \qquad (1)$$

Sign of k: if positive is closed, if negative open (and $\sin\theta \leftrightarrow \sinh\theta$).

Constituents of the Universe determine its expansion:

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{8\pi G}{3}\rho = -\frac{k}{a^2}, \qquad (2)$$

$$H \equiv \frac{\dot{a}}{a}, \qquad M_{\text{Planck}}^{-2} = 8\pi G, \qquad \rho_{\text{crit}} = 3H^2 M_{\text{Planck}}^2, \qquad (3)$$

$$\rho_{\text{non-rel}} \sim a^{-3}, \quad \rho_{\text{rad}} \sim a^{-4}, \quad \rho_{\Lambda} \sim a^0, \quad \rho_{\text{curvature}} \equiv -3M_{\text{Planck}}^2 \frac{k}{a^2} \qquad (4)$$

$$\Omega_i \equiv \frac{\rho_i}{\rho_{\text{critical}}}, \qquad 1 = \Omega_{\text{matter}} + \Omega_{\text{rad}} + \Omega_\Lambda + \Omega_{\text{curvature}} \tag{5}$$

Acceleration of the Universe

$$\frac{d^2a}{dt^2} = -\frac{4\pi}{3}(\rho + 3p)a. \tag{6}$$

Acceleration: $p < -\rho/3$

Cosmological constant: $p = -\rho$

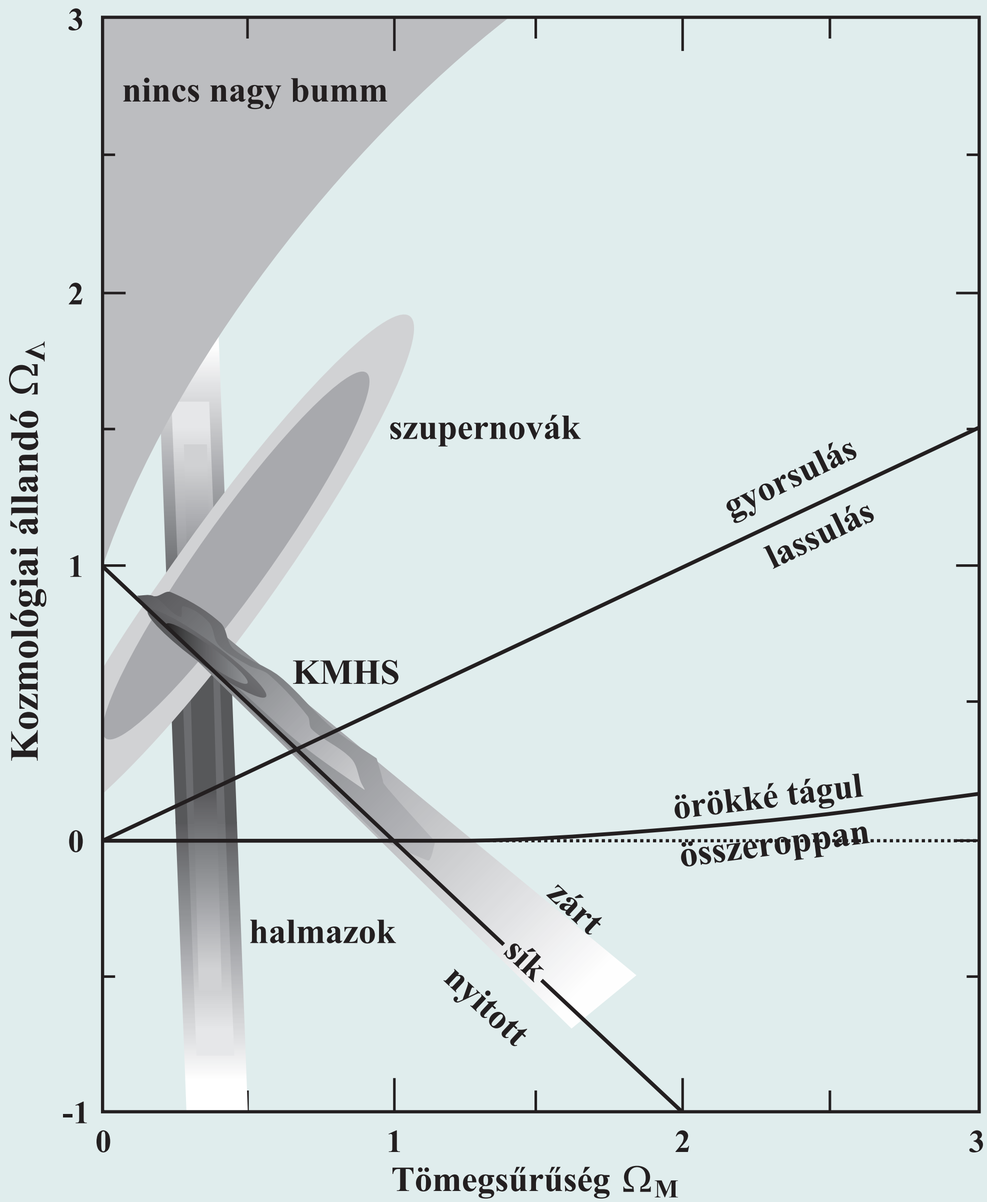

Figure 1: Az Univerzum energiasűrűségének három független módszerrel történt vizsgálatából kialakuló kép az $(\Omega_m, \Omega_\Lambda)$-síkon

Boltzmann-equation in the expanding Universe

One-particle phase space distribution $f(\mathbf{x},\mathbf{p},t)$:

$$\int \frac{d^3p}{(2\pi)^3} f(\mathbf{x},\mathbf{p},t) = n(\mathbf{x},t)$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{dx^i}{dt}\frac{\partial f}{\partial x^i} + \frac{dp^i}{dt}\frac{\partial f}{\partial p^i} = R(\mathbf{x},\mathbf{p},t)$$

R: Collision rate into and from phase space element $d^3x d^3p$

Assuming homogeneity in space and isotropy in momentum space:

$$\frac{\partial f}{\partial t} + \frac{dp}{dt}\frac{\partial f}{\partial p} = R(p,t)$$

Effect of expansion based on de Broglie relation $p \sim \lambda^{-1}$:

$$p(t)\lambda(t) = p(t)a(t) \times \lambda(t)/a(t) = \text{const.}$$

Consequence: $\dot{p}(t)a(t) + p(t)\dot{a}(t) = 0, \qquad \frac{dp}{dt} = -Hp$

$$\int \frac{d^3p}{(2\pi)^3} p \frac{\partial f}{\partial p} = -3n(t), \qquad \int_p \left(\frac{\partial f}{\partial t} - Hp \frac{\partial f}{\partial p} \right) = \frac{1}{a^3} \frac{d(na^3)}{dt}$$

$$R(t) = \int_{p_1} R(p_1, t) = \tag{7}$$

$$= \int_{p_1} \int_{p_2} \int_{p_3} \int_{p_4} (2\pi)^4 \delta(p_1 + p_2 - p_3 - p_4) |M(1 + 2 \leftrightarrow 3 + 4)|^2 (f_3 f_4 - f_1 f_2)$$

Approximate parametrization of Boltzmann-distribution:

$$f(E, t) \sim e^{-(\mu(t)+E)/T(t)} \sim f^{(0)}(T(t)) e^{-\mu(t)/T(t)}$$

$$n(t) = e^{-\mu(t)/T(t)} n^{(0)}(t)$$

Definition:

$$\overline{\sigma v} \equiv \frac{1}{n_1^{(0)} n_2^{(0)}} \int_{p_1} \int_{p_2} \int_{p_3} \int_{p_4} (2\pi)^4 \delta(p_1 + p_2 - p_3 - p_4) |M(1 + 2 \leftrightarrow 3 + 4)|^2 e^{-\frac{E_1 + E_2}{T(t)}}.$$

Rate equation for the time evolution of the density:

$$\frac{1}{a^3}\frac{d(n_1 a^3)}{dt} = n_1^{(0)} n_2^{(0)} \overline{\sigma v} \left(\frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} - \frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}} \right) \tag{8}$$

Similar equation can be written for the components 2,3,4.